

LXD RESEARCH

**ESSA**  
**Level 4**  
**Evidence**

# *Zaner-Bloser Kickstart:* *Number Sense* **Foundational** **Research Paper**

RESEARCH-BASED DESIGN & LOGIC MODEL



*Zaner-Bloser Kickstart: Number Sense* enhances students' foundational mathematical thinking to deepen their conceptual understanding of numbers (number sense), bolstering proficiency in grade-level skills and strengthening their identity as mathematicians to build community, curiosity, and confidence in math learning.

## *Zaner-Bloser Kickstart: Number Sense*

### Foundational Research Guide:

#### Research-based Design of *Zaner-Bloser Kickstart: Number Sense*

*Zaner-Bloser Kickstart: Number Sense enhances students' foundational mathematical thinking to deepen their conceptual understanding of numbers (number sense), bolstering proficiency in grade-level skills and strengthening their identity as mathematicians to build community, curiosity, and confidence in math learning.*

### Outline

- I. How Can This Research Foundations Paper Be Used?
- II. Key Points
- III. *Zaner-Bloser Kickstart: Number Sense* Logic Model
- IV. The Goal of *Zaner-Bloser Kickstart: Number Sense* Instruction
  - A. Short-Term Outcomes: Enabling students to understand and represent quantities efficiently, flexibly, and accurately to create fluent number sense in students
  - B. Long-Term Outcomes: Increasing access to building students' community, curiosity, and confidence in learning grade-level material
- V. How Does *Zaner-Bloser Kickstart: Number Sense* Help Students Build Number Sense Fluency? Each section includes a definition, a summary, and a deep dive into the topic.
  - A. **Engagement:** students positively engage with the learning opportunities
  - B. **Deepen Conceptual Understanding:** With a focus on developing automaticity and mastery of each concept at the concrete level, students more accurately and flexibly manipulate numbers and deepen their number sense.
  - C. **Advance to proficiency:** gaps are addressed by focusing on key skills and using concrete-representational-abstract progression for students to master challenging mathematical concepts with ease, advancing toward grade-level proficiency
  - D. **Strengthen math identity:** by knowing how to think and talk about numbers, student confidence increases to build their identity as a mathematician
- VI. Are you ready to improve your students' number sense?

## How Can This Research Foundations Paper Be Used?

This research paper aims to shed light on the innovative strategies, pedagogical approaches, and empirical evidence that inform this program’s design. *Kickstart: Number Sense* is designed to foster number sense in students from kindergarten through second grade. Each section details how *Kickstart: Number Sense* enhances students’ foundational mathematical thinking by deepening their conceptual understanding of numbers (number sense), bolstering proficiency in grade-level skills and strengthening their identity as mathematicians to build community, curiosity and confidence in math learning.

## Key Points

- Students engage in partner-talk and multimodal activities with physical movement and visual cues that support enhanced cognitive function and active engagement.
- Students develop number sense through a focus on specific prerequisite numerical skills chosen to support the development of foundational mathematical understanding
  - Four categories of skills: symbolic and non-symbolic number sense (basic magnitude comparison), understanding mathematical relations, counting skills, and basic skills in arithmetic.
- Students engage in “retrieval practice” by pulling out information and practicing what they’ve already learned.
- Students revisit topics over time to deepen their learning and improve retention as a result of the spiraled approach of interleaved and spaced practice.
- Instruction follows the natural progression of student learning from concrete to representational to abstract.
- Students receive systematic and explicit instruction based on developmentally sequenced activities.
- Instructors utilize integrated assessments and progress monitoring tools that closely match content and target skills to provide a scaffolded approach to support students in mastering challenging concepts.
- Students develop the confidence and understanding to talk and think about numbers, boosting their self-efficacy and establishing their identity as mathematicians.

## What Is Number Sense, and Why Does it Matter?

Humans inherently rely on an approximate number system (ANS) to provide basic intuitions of quantity without relying on language or symbols. This nonverbal system, which we refer to as number sense, is not dependent on explicit instruction. In contrast, humans develop a more formal, symbolic system of mathematics that provides for computations of exact quantity through explicit instruction consisting of symbolic representations, quantitative concepts, and formalized computational operations (Bonny & Luarenco, 2013). Evidence points to the functional role that number sense plays in symbolic arithmetic, laying the groundwork for enhanced performance in speed and accuracy of symbolic mathematics (Hyde et al., 2014) when provided in instructional settings.

Therefore, we understand that number sense, the flexible, nonverbal understanding of numbers and their relationships, is the cornerstone for mathematical proficiency. The development of number sense relies on a strong foundation of these prerequisite skills that are essential for a strong grasp of mathematical concepts. Number sense can be described as good intuition about numbers and their relationships. It develops gradually due to exploring numbers, visualizing them in various contexts, and relating them in ways that are not limited by traditional algorithms (Howden, 1989). These primary abilities form the basis for secondary symbolic number competencies, and early verbal number competencies are necessary for extending knowledge with small numbers to knowledge with larger numbers as well as for learning school-based mathematics (Jordan et al., 2010).

What does number sense look like in the math classroom? Children with strong number sense can:

- visualize and talk comfortably about numbers,
- take numbers apart and put them back together in different ways,
- compute mentally,
- relate numbers to real-life problems by connecting them to their everyday world, and
- use numbers and quantitative methods to communicate, process, and interpret information (McIntosh et al., 2005; Jordan et al., 2009).

Research also indicates that early number sense predicts school success more than other measures of cognition such as verbal, spatial, or memory skills or reading ability (Jordan et al., 2007) and that foundational skills in numbers and operations may set the stage for reading skills (Frye et al., 2013). Furthermore, number sense develops as students deepen their understanding of the size of numbers, develop multiple ways of thinking about and representing numbers, use numbers as referents, and develop accurate perceptions about

# Kickstart: Number Sense Foundational Research Paper


the effects of operations on numbers (National Council of Teachers of Mathematics, 2000). Children often demonstrate interest in math well before entering school, and in its Teaching Math to Young Children Practice Guide, The What Works Clearinghouse presents five recommendations to capitalize on this inherent interest and make their early learning experiences more engaging and beneficial:

**Table 2. Recommendations and corresponding levels of evidence**

Recommendation	Levels of Evidence		
	Strong Evidence	Moderate Evidence	Minimal Evidence
1. Teach number and operations using a developmental progression.		◆	
2. Teach geometry, patterns, measurement, and data analysis using a developmental progression.			◆
3. Use progress monitoring to ensure that math instruction builds on what each child knows.			◆
4. Teach children to view and describe their world mathematically.			◆
5. Dedicate time each day to teaching math, and integrate math instruction throughout the school day.			◆

Recommendation one emphasizes the importance of teaching numbers and operations using a developmental progression. To ensure effective instruction, it is imperative to identify the knowledge children already possess and build upon that knowledge to take them to the next developmental step. Developmental progressions, such as seen below, can inform instructors about which skills a child needs to work on in order to move forward (Frye et al., 2013). Based on this research, *Kickstart: Number Sense* uses a developmental progression of instruction that moves from the concrete to representational to abstract.

**Table 3. Examples of a specific developmental progression for number knowledge**



<p><b>Subitizing (small-number recognition)</b></p>	<p>Subitizing refers to a child’s ability to immediately recognize the total number of items in a collection and label it with an appropriate number word. When children are presented with many different examples of a quantity (e.g., two eyes, two hands, two socks, two shoes, two cars) labeled with the same number word, as well as <u>non-examples</u> labeled with other number words (e.g., three cars), children construct precise concepts of one, two, and three.</p> <p><b>A child is ready for the next step when</b>, for example, he or she is able to see one, two, or three stickers and immediately—without counting—state the correct number of stickers.</p>
<p><b>Meaningful object counting</b></p>	<p>Meaningful object counting is counting in a one-to-one fashion and recognizing that the last word used while counting is the same as the total (this is called the <u>cardinality principle</u>).</p> <p><b>A child is ready for the next step when</b>, for example, if given five blocks and asked, “How many?” he or she counts by pointing and assigning one number to each block: “One, two, three, four, five,” and recognizes that the total is “five.”</p>
<p><b>Counting-based comparisons of collections larger than three</b></p>	<p>Once children can use small-number recognition to compare small collections, they can use meaningful object counting to determine the larger of two collections (e.g., “seven” items is more than “six” items because you have to count further).</p> <p><b>A child is ready for the next step when</b> he or she is shown two different collections (e.g., nine bears and six bears) and can count to determine which is the larger one (e.g., “nine” bears is more).</p>
<p><b>Number-after knowledge</b></p>	<p>Familiarity with the counting sequence enables a child to have <u>number-after knowledge</u>—i.e., to enter the sequence at any point and specify the next number instead of always counting from one.</p> <p><b>A child is ready for the next step when</b> he or she can answer questions such as, “What comes after five?” by stating “five, six” or simply “six” instead of, say, counting “one, two, ... six.”</p>
<p><b>Mental comparisons of close or neighboring numbers</b></p>	<p>Once children recognize that counting can be used to compare collections and have number-after knowledge, they can efficiently and mentally determine the larger of two adjacent or close numbers (e.g., that “nine” is larger than “eight”).</p> <p><b>A child has this knowledge when</b> he or she can answer questions such as, “Which is more, seven or eight?” and can make comparisons of other close numbers.</p>
<p><b>Number-after equals one more</b></p>	<p>Once children can mentally compare numbers and see that “two” is one more than “one” and that “three” is one more than “two,” they can conclude that any number in the counting sequence is exactly one more than the previous number.</p> <p><b>A child is ready for the next step when</b> he or she recognizes, for example, that “eight” is one more than “seven.”</p>

Early basic whole number competencies, or understanding the meaning of numbers and number relationships, allow children to connect mathematical relationships, principles, and procedures (Gersten et al., 2005; Jordan et al., 2009). Weakness in general number competence is reflected in poorly developed counting procedures, slow fact retrieval, and inaccurate computation, which are all characteristic of general math difficulties (Geary et al., 2000; Jordan et al., 2003a, 2003b). Key areas of good number sense include counting, number knowledge, and arithmetic operations (Jordan et al., 2010).

- **Counting:** Children must be able to count fluently and understand the concept of one-to-one correspondence between objects and numbers. Accurate and efficient counting supports a strong understanding of number relations, ultimately leading to strong connections between a problem and its solution, reducing the need for rote memorization (Siegler et al., 1984, as cited in Jordan et al., 2009).
- **Number knowledge:** Recognizing and writing numbers accurately lay the groundwork for numerical representation, and the ability to visualize numbers and quantities in different contexts contributes to a deeper understanding of number relationships (Jordan et al., 2010).
- **Arithmetic operations:** Understanding basic operations such as addition and subtraction is fundamental, as number sense involves manipulating and comparing quantities (Jordan et al., 2010).

Language skills also play a significant role, as verbalizing and discussing mathematical ideas help reinforce number sense. Using open-ended questions and math conversation help children recognize math ideas in the world around them, gradually moving from informal representations and language to formal representations and math vocabulary as understanding grows (Frye et al., 2013). These prerequisite skills collectively provide the scaffolding for children to develop a strong foundation in number sense and excel in more advanced mathematical concepts.

*Kickstart: Number Sense* is a powerful educational resource that aids teachers in evaluating students' math proficiency. It offers progress monitoring forms and an assessment to pinpoint strengths and weaknesses. This assessment guides teachers in placing students in Stages A, B, or C, ensuring a tailored curriculum.

# Logic Model for *Kickstart: Number Sense*

## PROBLEM STATEMENT

Many students enter school with limited exposure to building number sense, a critical foundational mathematics skill. For students whose gaps are not filled by on-grade-level instruction, schools struggle with limited instructional time, insufficient resources, and a lack of opportunities for professional development. Instructors need to deliver engaging, targeted intervention for students to close skill gaps and build their confidence as mathematicians.

### RESOURCES

*What resources are or could be available?*

- Dedicated time for small-group interventions
- A provided model for assessment and intervention support to students
- Instructor access to a device, such as a laptop, and the internet for online instructional materials
- *Kickstart* student-facing learning content for building number sense
- *Kickstart* instructor guides with explicit teaching support so that any adult can lead an intervention

### STRATEGIES & ACTIVITIES

*What will the activities, events, and such be?*

- Students engage in partner talk and hands-on activities
- Targeted approach to the prerequisite number sense skills that are essential for math success
- Developmental progression of instruction that moves from the concrete to representational to abstract
- Concepts taught through multimodal activities and experiences that provide variety and active engagement
- Spiraled approach to instruction that incorporates both interleaving and spacing of content
- A structured, sequential approach to number sense development in the form of 20-minute lessons over 60 days
- Practice questions and immediate feedback to aid in learning
- Progress Monitoring tool to let instructors know what students have completed and learned, as well as where they struggled during the lessons

### OUTPUTS

*What are the initial products of these activities?*

- Instructors present more interactive lessons.
- Students talk about numbers as much as possible to gain confidence, recognize relationships between numbers, and develop flexibility in using them.
- Students explore numbers, visualize them in various contexts, and relate them to each other.
- Students engage with the materials through discussion and hands-on multimodal activities.
- Students' skills are corrected or reinforced through real-time feedback, which allows for differentiated instruction.
- Instructors use the included assessment to place students in the appropriate stage and progress monitoring tools to track data necessary for essential decision-making.

## SHORT-TERM AND INTERMEDIATE OUTCOMES

- Instructors build capacity in math instruction to support individual student growth.
- Instructors more successfully differentiate instruction for all learners.
- Students improve their math fact fluency.
- Students engage in math discourse, building language relationships and math understanding.
- Students find their math learning more relevant.
- Students retain and apply math learning in novel situations.
- Students are less likely to hesitate to complete math tasks.
- Students are more likely to be confident in their math abilities.

## LONG-TERM OUTCOMES AND IMPACTS

- Students build confidence in their ability to succeed in grade-level math, fostering self-efficacy so they can develop their independent working abilities within math skill development.
- The number sense knowledge gap between students who have different educational backgrounds is narrowed, fostering a greater sense of community when students can collaborate and communicate on similar levels.
- Students develop a sense of ownership over their mathematical skills, with the confidence to tackle increasingly difficult math problems.
- Students can process their learning, make connections, and build curiosity as they are presented with relevant, interesting contexts.

## ASSUMPTIONS

- Instructors engage in professional learning and relevant product training.
- The program is implemented fully and consistently.
- Administrators allow for a variety of support personnel to engage in tiered levels of support, not just classroom instructors.
- Activities are integrated and aligned with the overall math progression at the school and are not an isolated experience.

### The Goal of *Zaner-Bloser Kickstart: Number Sense Instruction*

*Kickstart: Number Sense* is built on evidence-based strategies to make early mathematics intervention effective. The program was created to help struggling students develop the confidence and understanding to talk and think about numbers fluently by incorporating interleaved practice. By taking this spiraled approach to instruction that involves revisiting topics over time, concepts can build on themselves as students gradually progress in learning, look for patterns, make connections, and experience productive struggle.

*Kickstart: Number Sense* is a K–2 small-group math intervention program designed to be taught by any instructor as every lesson is scripted, lists everything the teacher needs to know and provides all of the necessary materials within the classroom kit. It is a Tier 2 program meant to fill gaps in specific number sense holes and is designed to help students who enter school unprepared for the grade-level core curriculum. As an intervention, it is to be used at a separate time of day than the core curriculum and was written to be a 20-minute small-group pull-out, which fits flexibly into schedules. Additionally, the ready-to-use kits minimize preparation time, and the provided assessments help place students in the program while monitoring their progress.

Because *Kickstart: Number Sense* is an intervention program, it maintains a targeted focus on specific numerical skills that are crucial for children’s future mathematical development to allow struggling students to access the grade-level core curriculum. *Kickstart: Number Sense* addresses skills that fall into four categories (Aunio & Räsänen, 2016):

- symbolic and non-symbolic number sense (basic magnitude comparison)
- understanding mathematical relations
- counting skills
- basic skills in arithmetic

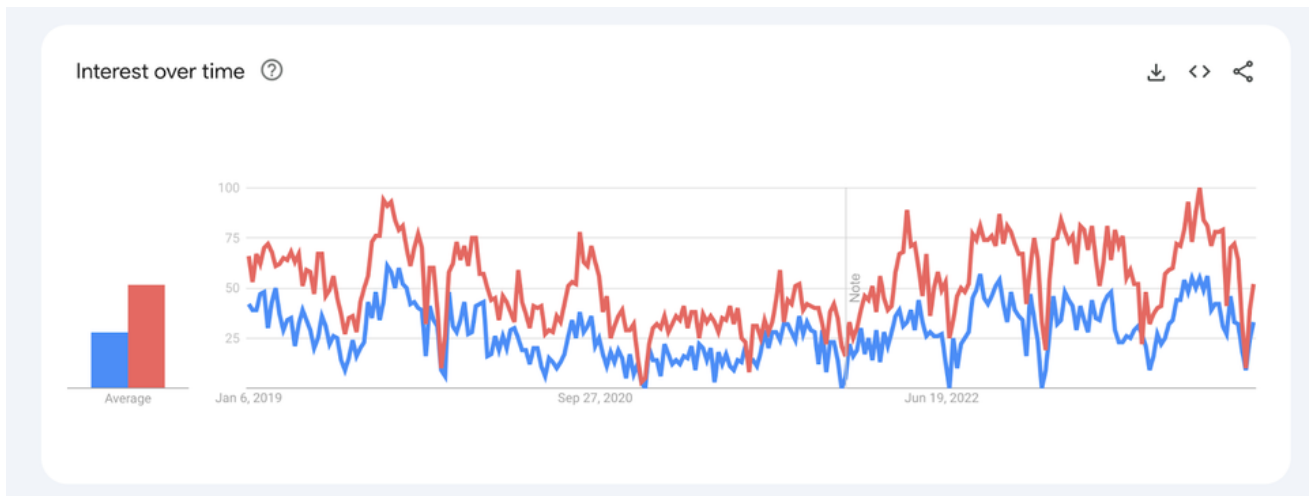
These early skills are intentionally and specifically targeted for intervention as they are crucial in developing a solid foundation for mathematics learning. Subitizing, hierarchical inclusion, and general number flexibility are built through experience with math tasks using objects and are not covered by the core curriculum. Successful, research-based math interventions follow systematic and explicit instruction based on developmentally sequenced activities (e.g., Sterner et al., 2019).

Researchers have uncovered that students enter school one to three years behind in mathematical thinking skills. Unfortunately, without targeted intervention, this trend tends to persist throughout elementary school. This phenomenon is still true today and has been exacerbated by the pandemic. The State of Learning report in 2023 showed that approximately two thirds of students in grades 1–5 placed on grade level before the pandemic; however, roughly half of grades 1–5 students have placed on grade level since the pandemic (Curriculum Associates, 2023). The report also highlights a lack of preparedness among teachers to address this issue and underscores how math intervention is not common in elementary school settings. In fact, based on data from Google Trends, which provides an unfiltered sample of actual Google web search requests, we can see how little, in comparison to reading intervention, math intervention is searched (Google Trends, 2015). Searching for an intervention for number sense is practically nonexistent.

## How does interest in math intervention differ from reading intervention?

Red = reading intervention

Blue = math intervention



*Kickstart: Number Sense* focuses on key numerical skills, utilizes effective teaching strategies, offers developmental progression from concrete to representational to abstract learning, and engages students through various multimodal activities. It is structured to cater to the diverse learning needs of students and provides a scaffolded approach to strengthen comprehension and mastery of mathematical concepts.

This program recognizes the significance and supports the growth of skills such as counting, number recognition, basic operations, spatial awareness, and language in fostering number sense in students. By incorporating an assessment and progress monitoring system, educators can determine students' mathematical strengths and weaknesses, track their growth over time, and make adjustments as needed.

### Set the Stage for “Aha” Moments!

*Kickstart* helps teachers create hands-on, active, and conversational environments where struggling students can discover connections and successfully transition from concrete to abstract thinking about numbers.



### Short Term Outcomes:

The short-term outcomes of the *Kickstart: Number Sense* math intervention program encompass a multifaceted approach aimed at fostering comprehensive mathematical development among students. Through engaging in multimodal activities, students will enhance their reasoning skills and cultivate a deep understanding of numbers by representing them in various ways. This program focuses on instilling a strong sense of numbers, enabling students to accurately gauge the magnitude of numbers and comprehend the impact of operations on them. Furthermore, the acquired number sense skills will be a foundation for advanced mathematical thinking. As a result, students will experience accelerated growth in their readiness for grade-level mathematics, concurrently bolstering their confidence in executing mathematical tasks and fostering their ability to retain and apply their math learning in novel situations. The program also aims to fortify students' ability to articulate mathematical concepts, facilitating effective communication. Simultaneously, teachers will benefit from the program by becoming adept at delivering tailored interventions to support individual student growth, differentiating instruction effectively, and utilizing progress monitoring tools to inform essential decision-making processes.

## Long Term Outcomes:

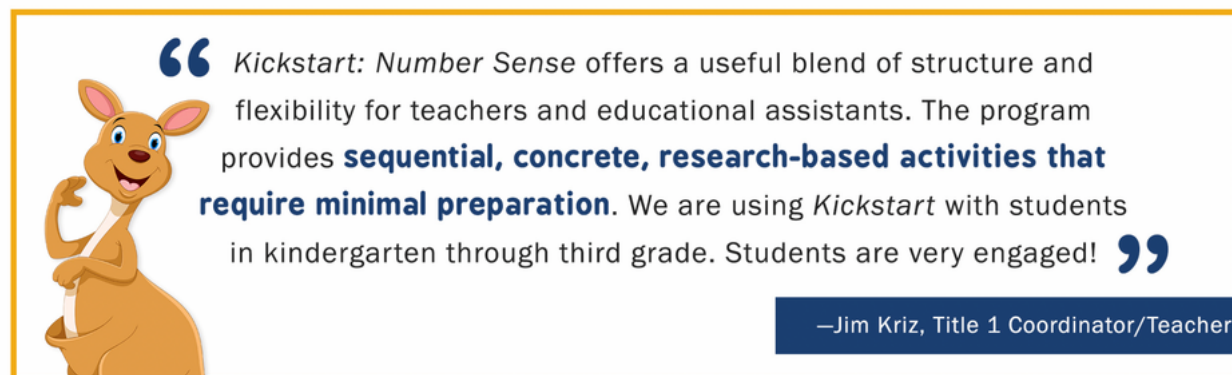
The long-term outcomes of the *Kickstart: Number Sense* math intervention program extend beyond immediate skill development, aiming to cultivate enduring changes in students' attitudes and approaches to mathematics. Students will develop a lasting sense of self-efficacy by building confidence in their ability to succeed at grade-level math, fostering independence in their mathematical skill development. Additionally, the program aspires to narrow the number sense knowledge gap between students with diverse educational backgrounds, promoting a greater sense of community through collaborative and communicative efforts. Over time, students are expected to develop a strong sense of ownership over their mathematical skills, identifying themselves as owners of their math, with the confidence to tackle increasingly challenging problems. Moreover, the program encourages students to process their learning, make meaningful connections, and nurture curiosity by presenting mathematical concepts within relevant and interesting contexts.

## How Does *Zaner-Bloser Kickstart: Number Sense* Help Students Learn?

### Engaging Students in Learning

#### *Why does engagement in learning matter?*

Engagement in learning foundational math skills is crucial for fostering a positive attitude, intrinsic motivation, and understanding of mathematical concepts. Engaged emotions, such as interest and enthusiasm, fuel engaged behaviors, such as effort and persistence (Skinner et al., 2008). Furthermore, engagement involves both behavioral and emotional aspects, representing active, purposeful, adaptable, constructive, persistent, focused, and emotionally positive interactions within the academic context with both social and physical surroundings (Skinner et al., 2008). Active participation in hands-on, multimodal activities enhances the retention and application of knowledge, developing critical problem-solving skills (Snyder et al., 2017).



“ *Kickstart: Number Sense* offers a useful blend of structure and flexibility for teachers and educational assistants. The program provides **sequential, concrete, research-based activities that require minimal preparation**. We are using *Kickstart* with students in kindergarten through third grade. Students are very engaged! ”

—Jim Kriz, Title 1 Coordinator/Teacher

Positive early experiences contribute to a confident math identity, reducing math anxiety and sustaining long-term interest in the subject and its applications (Aguirre et al., 2013). Children with initial confidence in their abilities and a positive math identity approach learning tasks in manners that result in greater success, reinforcing their initial optimism. Conversely, children with low efficacy tend to avoid challenges or participate in tasks with minimal effort, leading to a lack of success, solidifying their initial self-doubts (Skinner et al., 2008). Additionally, other classroom dynamics involve the instructor. Children who demonstrate higher levels of engagement tend to receive more instructor involvement, whereas disaffected students are more likely to find that instructors withdraw support or, conversely, become more controlling over time (Skinner & Belmont, 1993 as cited in Skinner et al., 2008).

To highlight instructor influence more deeply, math-anxious elementary school teachers, tend to negatively affect their students (Beilock et al., 2018). This also continues into secondary math education, severely undermining students' ability to acquire the mathematical knowledge needed for the workplace (Ramirez et al., 2018). Because *Kickstart: Number Sense* is scripted, provides all necessary materials, and is designed for easy implementation, instructor math anxiety can be reduced, thereby potentially reducing math anxiety and increasing engagement among students.

---

### *How Does Kickstart: Number Sense ENGAGE Students in Learning?*

---

*Kickstart: Number Sense* teaches concepts through multimodal activities and experiences that provide variety and active engagement. Each activity is designed to involve students physically, mentally, and emotionally while fostering a deep understanding of concepts. By encouraging students to actively participate and move as they learn, *Kickstart: Number Sense* creates a more effective and engaging learning experience, ensuring that students not only grasp mathematical concepts but also retain and apply them more effectively.

## DEEP DIVE INTO ENGAGEMENT

**Multisensory learning activities** are integrated into lessons to support numerical skills development. Subitizing and estimating are the two most important skills kids need in preschool to predict mathematical success (vanMarle, 2015). Subitizing is the ability to instantly recognize the number of objects in a group without the need for counting. Estimating is the ability to provide an approximation of the value, quantity, or number in mathematical situations. *Kickstart: Number Sense* encourages the mental strategies of subitizing and estimating by incorporating an element of speed, which challenges students to find strategies other than counting and provides an opportunity for students to reflect on and share their strategies (Presmeg, 1986; Mason, 1992 as cited in Way, 2011).

In *Kickstart: Number Sense*, students practice subitizing by viewing displays and reporting back the number of dots they see. For example, during Day 1 of Stage A, students begin by viewing a card with 2 dots for three seconds. When the card is no longer displayed, students are asked to visualize what the dots look like, and without speaking, students show how many dots they think were on the card using their fingers. After students have this individual opportunity, the teacher and students collectively count together as a group to determine the number of dots.

## Subitizing Subitizing

### • Day 1 Subitizing Dot Cards

**T: Today I'm going to show you a card with dots. Your job is to decide how many dots are on my card but keep your good ideas silently in your head. When you think you know, give a thumbs up.**

*Hold up the Day 1 Subitizing Dot Card with two dots for three seconds. Put the card face down while students think for a moment.*

**T: Close your eyes and try to remember what you saw. We call this visualizing. Can you visualize what those dots looked like?**

*Visualization is an excellent strategy, but one that often needs to be explicitly taught.*

**T: Without saying anything, hold up your guess on your fingers. How many dots?**

*Check that everyone has 2. If one or two students do not, this is not a big problem. Those students will get better at this activity through the weeks. If most students do not, show the card again for three more seconds asking them to try again.*

**T: There are 2 dots. Let's count the dots together.**

*Hold up the card again so everyone can see it. Point as you count.*

*Students may have "just seen" 2. They may have been able to subitize the number. That is what we are hoping for, but not expecting in an intervention setting. Research suggests that 5-year-olds can subitize to 5, but this is infrequent for students struggling in math. Daily practice is vital.*

**T: I'm going to show one more card with dots. Remember, your job is to decide how many dots there are on my card while keeping the answer in your head. When you think you know, give a thumbs up.**

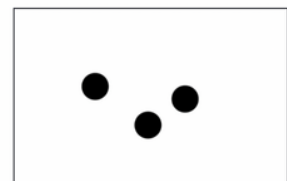
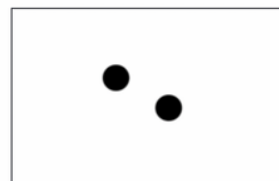
*Hold up the Subitizing Dot Card of three dots for three seconds. Put the card face down while students think for a moment.*

**T: Without saying anything, hold up your guess on your fingers.**

*Check that everyone has 3.*

**T: There are 3 dots. Let's count those 3 together.**

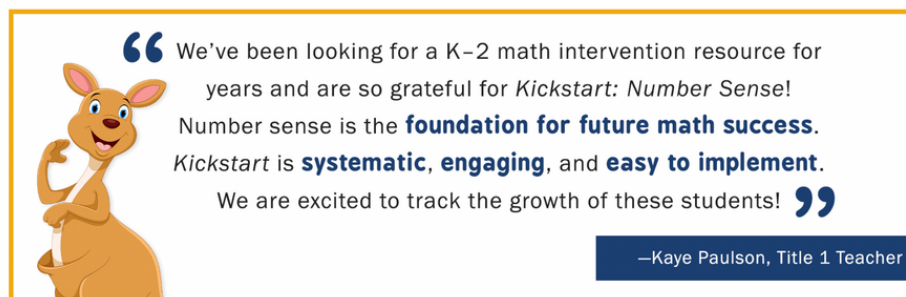
*Count aloud as you touch the dots. Take turns with students having them touch and count the dots.*



## Kickstart: Number Sense Foundational Research Paper

If students can “just see” 2 dots without counting, they are relying on the strategy of subitizing. Students complete practice with different numbers of dots for reinforcement to improve their ability to tap into this strategy. In this same lesson, students simultaneously participate in singing and movement exercises as they count to 20, applying research findings highlighting the memory-enhancing benefits of motor actions. The program’s emphasis on motor actions aligns with the understanding that physical engagement can enhance memory retention and cognitive processing (Madan & Singhal, 2012). Students engage in songs, jump on number lines, play games, and engage with hands-on manipulatives. These visual cues and physical activity support enhanced cognitive function, improved self-esteem, reduced stress, and increased positive academic outcomes (Snyder et al., 2017).

With the Number of the Day activity, students match quantities by using their fingers to show the day’s number, identify the number on a “Number Path” of numbers 1–10, and use cubes to represent that number and place them on the Number Path. These activities not only make learning enjoyable but also allow students to talk comfortably about numbers and share their thinking aloud. The program promotes a positive math identity by creating an environment where students actively participate in their number sense journey. The use of a plush friend named Cota Kangaroo adds a friendly and supportive element to the learning process.



**Opportunities to connect with peers** through partner talk is pivotal to the program in order to foster community, establish math identity, learn from others, and develop language skills, and because we learn math through the language center of our brain (Sousa, 2015). Hattie refers to classroom discussion (aka math discussion) as a 0.82 effect size. Students who process math through language are deeper conceptual thinkers and better at owning the math themselves. When students use *Kickstart: Number Sense*, they are still developing their language skills, either because they are new to the language or because they are primary students, and subsequently need the practice with vocabulary and developing their arguments.

# Kickstart: Number Sense Foundational Research Paper

*Kickstart: Number Sense* incorporates practice with language and partner talk throughout the program, which supports Math Practice #3: Construct viable arguments and critique the reasoning of others. Opportunities for math language practice include prompts for students to share what they wonder and what they notice with others, answering the question of the day, and providing reasoning for their responses. Students are able to process language through discussion with their “Sun and Moon” partners, which provides an opportunity for every student to talk and process the language, allowing them to retain the math concepts they need. These opportunities for partner talk are laced throughout each day of each stage to help students gain math identity, math vocabulary, and confidence in explaining their thinking.

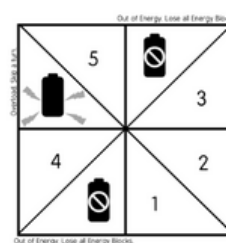
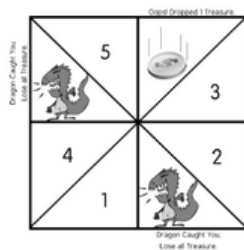
## Teachers Praise *Kickstart's* Accessibility

*Kickstart* answers the question on every primary teacher's mind: How do I reach my students who struggle in math because they lack the number sense needed to be successful in a conventional math program? This straightforward intervention does not require a teacher to master yet another curriculum, nor does it need an extended period of time to implement. Its short, simple lessons are easily accessible to both teachers and students.

—Karee Hall, Teacher



**Motivational supports** are embedded within *Kickstart: Number Sense* to encourage students to continue in their learning efforts. These elements are designed to create a positive and enjoyable learning experience, fostering a sense of motivation and curiosity about numbers. According to the Self-Determination Theory, students are more motivated when they feel autonomous, competent, and connected while learning (Ryan & Deci, 2000). In the first days of the program, students are carefully guided how to use specific math tools and later given the freedom to choose the math tool that works best for them. Embedded games such as “Dragon’s Treasure” and “Charge It Up” motivate students to learn number sense through play. The explicit, systematic approach of *Kickstart: Number Sense* bolsters competence in math and is intentionally designed to promote students’ autonomy and advance their skills.



### Deepening Students' Understanding of Number Sense

#### *Why does deepened mathematical understanding matter?*

Number sense, the understanding of numbers and their relationships, is the cornerstone upon which mathematical proficiency is built. The ability of a child to understand and work with sets of numbers in first grade is a strong predictor of their future success in both secondary school math and day-to-day living. In a study of 180 seventh-graders conducted by the University of Missouri, researchers found that, “those who lagged behind their peers in a test of core math skills needed to function as adults were the same kids who had the least number sense or fluency way back when they started first grade” (Neergaard, 2013).

Number sense extends beyond just knowing numbers—it’s about seeing patterns, making informed estimations, and engaging in strategic problem-solving. When there are gaps in number sense, students may struggle to grasp more advanced mathematical concepts, leading to difficulties in problem-solving, critical thinking, and overall mathematical proficiency. Research indicates that a deepened, early understanding of mathematical relationships predicts school success more than other measures of cognition, such as verbal, spatial, or memory skills or reading ability (Jordan et al., 2007).

---

#### *How does Kickstart: Number Sense deepen students' understanding?*

---

*Kickstart: Number Sense* specifically targets foundational mathematics skills, including symbolic and non-symbolic number sense, understanding mathematical relationships, counting skills, and basic arithmetic. The program employs research-based strategies, hands-on multimodal activities, and diagnostic assessments to foster a strong foundation in mathematics to develop automaticity. By incorporating interleaved and spaced practice, which involves mixing up problems relating to different concepts and deliberate intervals between repetitions, the program aligns with cognitive science and supports long-term retention and mastery of mathematical concepts.

### DEEP DIVE INTO DEEPENED MATHEMATICAL UNDERSTANDING

*Kickstart: Number Sense* incorporates interleaved and spaced practice to deepen learning and promote better retention, transfer, and long-term mastery. Mass, or blocked practice, is characterized by introducing and practicing one concept for a long period of time. Research has shown that “mastery and long-term retention are much better if you integrate practice than if you mass it” (Brown et al., 2014). The strategy of mixing up problems, or interleaving, for multiple concepts to be learned can significantly increase math learning and retention. When these concepts are mixed, students must engage in “retrieval practice” by thinking carefully, pulling information out, and practicing what they know (Agarwal & Agostinelli, 2020). Although mixing up practice may appear counterproductive, it significantly improves long-term performance. The productive struggle created from mixed practice helps the brain create more efficient neural pathways (Sriram, 2020).



Research shows that students who space out their learning outperform students who try to learn in longer sessions, even if spacing results in less total time on task. You can use the concept of spacing by touching on important concepts multiple times a week in brief, regular sessions.

(Sriram, R., 2020)



Because *Kickstart: Number Sense* is an intervention program, it does not include coverage of every core mathematical concept. Instead, the program focuses on the specific numerical skills crucial for children’s future mathematical development to allow struggling students to access the grade-level core curriculum. These skills fall into four categories: symbolic and non-symbolic number sense (i.e., basic magnitude comparison); understanding mathematical relations; counting skills; and basic skills in arithmetic (Aunio & Räsänen, 2016). These early skills are targeted for intervention because they are critical for developing a strong foundation for mathematics learning.

In the context of *Kickstart: Number Sense*, interleaved practice ensures that students encounter a diverse set of numerical problems within a given session. Because the program has a targeted focus on specific skills, rather than coverage of every possible skill, plenty of time is devoted to interleaved practice across different skills. For example, students may not only focus on counting, but also switch between tasks such as symbol recognition, mathematical relations, and basic arithmetic.

This intentional mixing challenges learners to actively retrieve and apply their knowledge, reinforcing connections between different mathematical concepts. *Kickstart: Number Sense* incorporates spaced practice by structuring learning sessions and activities with deliberate intervals between repetitions of specific numerical skills. This approach aligns with cognitive science research, which suggests that spacing out learning over time enhances long-term retention (Karpicke & Roediger III, 2007). For instance, *Kickstart: Number Sense* may revisit certain number sense skills in subsequent lessons, reinforcing learning at spaced intervals to solidify understanding. Using interleaved and spaced practice in *Kickstart: Number Sense* helps students deepen their understanding of the material to develop a strong foundation in mathematical concepts.

### Helping students quickly advance their proficiency in grade-level skills

#### *Why does grade-level proficiency matter?*

Grade-level proficiency provides a benchmark to ensure that students acquire the essential mathematical skills deemed appropriate for their grade level. Proficiency at each grade level signifies that students have grasped foundational concepts, setting the stage for more advanced learning in subsequent years. In a study of 180 seventh-graders conducted by the University of Missouri, researchers found that, “those who lagged behind their peers in a test of core math skills needed to function as adults were the same kids who had the least number sense or fluency way back when they started first grade” (Neergaard, 2013).

Demonstrating grade-level proficiency and having a strong foundation in mathematics at each stage of education contribute to a cumulative understanding of the subject, enabling students to tackle increasingly challenging math concepts.

**“Mathematical concepts build on each other throughout the school years. A strong foundation allows students access to later mathematical concepts.”**

~Jen Hunt, Author of *Kickstart: Number Sense*

---

### *How does Kickstart: Number Sense help students gain proficiency in grade-level skills?*

---

*Kickstart: Number Sense* is a focused intervention for students who are struggling with grade-level math skills. By specifically addressing number sense, which is foundational to overall math proficiency, *Kickstart: Number Sense* provides a targeted approach to bridge gaps in understanding and quickly bring students up to speed. The program emphasizes hands-on learning experiences, including using manipulatives and multimodal activities. This format gives students a concrete understanding of mathematical concepts, making abstract ideas more accessible. Focusing on the concrete to representational to abstract approach accelerates comprehension and retention of grade-level math skills.

#### DEEP DIVE INTO GAINING PROFICIENCY

A critical framework in education is the understanding of the appropriate developmental progression of instruction, an approach that emphasizes instruction should move from concrete to representational to abstract. This instructional approach recognizes the diverse learning needs and cognitive development of students. Successful, research-based interventions follow systematic and explicit instruction based on developmentally sequenced activities (Stern et al., 2019). The *Kickstart: Number Sense* program draws on a large body of research to understand the general progression of students' strategies as they are developing mathematics skills (e.g., Piaget, 1952; Bruner, 1964, 2004; Sousa, 2008; Hauser, 2009; Carpenter et al., 2015). Research tells us that effective mathematics instruction encourages students to establish connections among different mathematical representations, enhancing their comprehension of mathematical concepts and procedures. Additionally, these representations serve as valuable tools for problem-solving (Leinwand, 2014).



Students who struggle with number sense frequently struggle with the concept that numbers are not static. They can be big or small, they have relativity and magnitude.

(Shumway & West, 2011)



*Kickstart: Number Sense* was designed with the natural developmental progression of student learning in mind, moving from concrete to representational to abstract. In this approach, learners begin with concrete experiences, which involve hands-on activities with tangible objects, such as fingers and manipulatives. This stage allows them to build a strong foundation of understanding by creating mental models of numbers and mathematical concepts through direct manipulation and sensory exploration. Importantly, students with limited interaction with concrete math concepts before starting school need supplemented time with physical number sense experiences like those *Kickstart: Number Sense* provides. As students progress to the representational stage, they transition to using visual representations, such as diagrams, models, or drawings, to connect their concrete experiences to more abstract concepts. This phase helps learners bridge the gap between the tangible and the conceptual, facilitating deeper comprehension. Finally, in the abstract stage, students are ready to work with abstract symbols and equations, applying their solid foundation of knowledge to solve complex problems.

# Kickstart: Number Sense Foundational Research Paper

This developmental progression acknowledges that learning is not one-size-fits-all and provides a scaffolded approach to effectively support students in mastering challenging concepts. This progression repeats as children learn larger numbers and foundational number sense in kindergarten is a strong predictor of math achievement in elementary school (Jordan et al., 2012). By applying a concrete to representational to abstract progression of instruction, *Kickstart: Number Sense* supports a holistic and effective approach to teaching, promoting understanding, engagement, and inclusivity in mathematics education.

**Assessment and progress monitoring tools** that are customizable and flexible help teachers quickly gauge where students are in their learning, helping to identify students' individual strengths and weaknesses, eliciting various types of recall/retrieval, and appealing to a variety of student cultural backgrounds (Agarwal & Bain, 2019). Effective formative assessment tools also help teachers quickly diagnose where students are having difficulty and provide specific, immediate, targeted feedback to support continued learning (Brookhart, 2017). Program-provided assessments typically include items that closely match the content and skills fostered in particular interventions.

On the contrary, standardized assessments may prioritize content and skills that are not necessarily covered in an intervention program (Wang et al., 2016).

Kickstart: Number Sense

Daily Progress Monitoring SAMPLE

**Daily Progress Monitoring - SAMPLE**  
 Use this form to track student progress through the stage.  
 Pick any skills to track or track them all.  
 Use check marks to show achievement of understanding.

Student Name  
 Romel

Day or Date	Adding Within 1	Counting On	Decomposing Numbers	Doubles	Estimating	Near Doubles	Partners to 10	Place Value	Skip Counting	Subitizing	Teen Number
Monday 3/5	✓		X	✓							✓
Tuesday 3/6	✓		X	✓							✓
Wednesday 3/7	✓		X	✓							✓
Thursday 3/8	○		○	○							○
Monday 3/12	✓		✓	✓							✓
Tuesday 3/13	✓		X	✓							✓

In this sample, an instructor is working through Stage B. This instructor chose to track the student understanding on four specific skills categories each day.

The open circles represent Romel's absence on Thursday the 8<sup>th</sup>. The check marks represent that Romel was able to work at the expected level during that activity that day. The "x" represents his inability to comprehend the concept that day. The horizontal line denotes that skill was not addressed in that day's lesson.

The blank form in the following pages can be copied multiple times for all students in each stage's group.

Using the included *Kickstart: Number Sense* diagnostic assessment, teachers begin the intervention by assessing students' mathematical understanding to determine individual starting points in the program. This assessment identifies strengths and gaps, guiding instruction tailored to each student's needs. The program's model ensures a systematic and targeted approach with continuous assessment, allowing teachers to adapt instruction based on individual progress.

*Kickstart: Number Sense* also has built-in daily progress monitoring tools that help instructors keep track of student understanding and where to focus their instruction. Instructors can mark whether the skill was addressed that day and whether the student could work at the expected level during the activity. This brief checklist provides a quick visual for instructors to refer back to when starting each lesson. Intervention products that include frequent, low-stakes assessment moments help teachers easily see what kids are picking up on a daily basis. More than anything, these progress monitoring tools help the teacher keep track of everyone, every day. With the *Kickstart: Number Sense* progress monitoring sheets, at the end of the week, if instructors see that a student is still not getting something, they can plan accordingly for the following lessons. Without this sort of visual tracking tool, it is incredibly challenging for a teacher to keep track of all this in their head. Because *Kickstart: Number Sense* breaks down the learning on the weekly sheets into very specific skills, instructors can mark where students are to best help them build all the necessary skills.

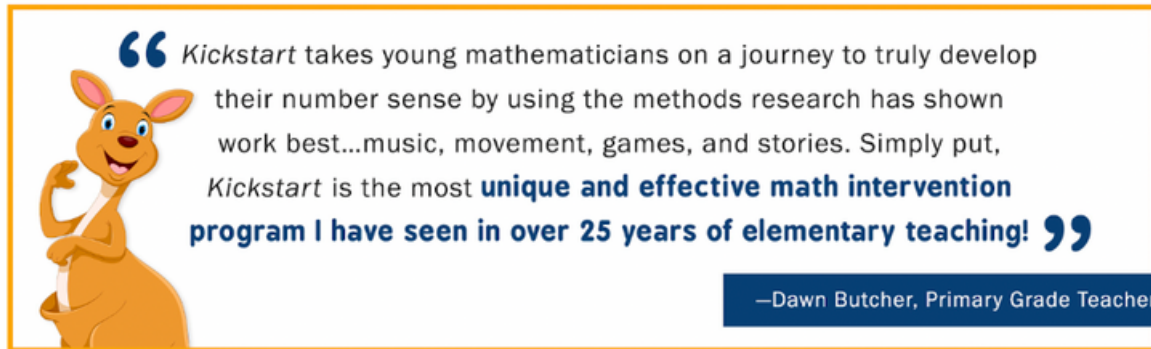
### Strengthening students' identity as mathematicians

#### *Why does a student's identity as a mathematician matter?*

In order to support the next generation of math learners, it is important that educators support students' identities as mathematicians, helping students buy into the idea that they possess their own mathematical abilities. By making math about more than just numbers and incorporating real-world applications to demonstrate relevance and creativity, educators can play a critical role in shaping students' mindset, perseverance, and sense of belonging. In this way, students are guided away from a fixed mindset, where they perceive themselves as not being naturally inclined toward math, to a growth mindset, emphasizing that effort and determination lead to success.

## How does Kickstart: Number Sense strengthen math identity?

The *Kickstart: Number Sense* intervention program is designed to strengthen students' identity as mathematicians by encouraging student ownership of their learning and by fostering a positive learning environment. The program's small-group setting encourages collaboration and community-building. *Kickstart: Number Sense* addresses the needs of underrepresented students, fostering inclusivity and valuing each student's contributions.



### DEEP DIVE INTO STRENGTHENING MATH IDENTITY

*Kickstart: Number Sense* encourages students to take ownership of their learning. Having a sense of ownership over their work instills confidence and motivates students to actively engage in their mathematical development, encouraging the belief that they have the capability to succeed in math. Math identity is framed in capability and confidence – when young learners have confidence and belief in their math abilities, they play an active role in the learning process, demonstrate improved retention and understanding, and are prepared for success in future, more challenging math environments (SanGiovanni et al., 2021).

# Kickstart: Number Sense Foundational Research Paper

Opportunities for students to share and hear about others' experiences with math can help learners connect and reflect on the importance of math in their own lives. With *Kickstart: Number Sense*, educators have the ability to leverage Cota Kangaroo to encourage ownership and make math more relevant. By incorporating the math story of Cota, a kangaroo who wants to learn but struggles sometimes, makes mistakes, and needs the help of others, students can better learn to accept their own math journey through learning and develop greater perseverance. Each student will have a unique math story that includes experiences that have helped shape who they are as learners. By having students reflect on their math journey, they may be able to illustrate their own perceptions of why it is important to learn mathematics (SanGiovanni, 2021). When students uncover for themselves the importance of math, they develop intrinsic motivation and are empowered to approach math challenges with confidence and a proactive attitude. Intrinsically motivated individuals will persist at a task and thus be more likely to achieve (Garon-Carrier et al., 2016).

Students will need a partner most days. In this program they are referred to as "Sun" and "Moon" partners. It is most beneficial when students have the same partner each day for at least a few weeks before changing partners. If there is an odd number of students, read ahead and decide if the activity can accommodate three or if you become the partner of the last student. If there are three students in a group, you have Sun, Moon, and Stars.

## Counting Counting

• 10 cubes per partnership

*Give the cubes to the Suns at first.*

**T: Suns, you have 10 cubes. Pick an amount to put in front of your Moons. Your Moons will count them by moving them from one side to the other, making a nice line. Remember when we did that a few days ago? If your Moon counts correctly, they get to go next, putting some in front of you. If they count incorrectly, you smile politely and ask, "Are you sure about that?" That is the signal they need to try again. It's okay to try again! Mistakes are where learning happens!**

*Monitor carefully to ensure students are counting correctly, and if not, their partner is catching the mistake. Be sure partners are switching jobs and following the guidelines you have set.*

Kickstart: Number Sense Stage A

*Kickstart: Number Sense* also creates a positive and inclusive learning environment where students feel comfortable discussing and engaging with numbers. *Kickstart: Number Sense* is designed for small groups of two to six students. A small-group setting of this size encourages peer interaction, providing a supportive context for students to share their thoughts, ideas, and problem-solving strategies. This collaborative atmosphere contributes to a sense of belonging and builds confidence in students' ability to navigate foundational mathematical concepts.

Establishing this sense of belonging is crucial for underrepresented students such as language learners and Black and Latinx students (Miller-Cotto & Lewis Jr., 2020). Creating an environment that acknowledges and respects the cultural backgrounds of students from diverse communities makes students feel that their cultural identities are valued, reducing the risk of marginalization and promoting a positive educational experience. By making mathematics relevant for Black, Latinx, and Multilingual learners, educators create a social process that enables students to understand how mathematics plays a role in their everyday lives, communities, and interests and further enables them to see themselves as mathematical thinkers and doers (Miller-Cotto & Lewis Jr., 2020).

Belongingness also directly and positively impacts multilingual learners, who often become isolated by linguistic and cultural differences. Feeling a sense of exclusion, being seen as culturally dissimilar, and having anxiety around oral communication (Finley, 2018; Beck & Malley, 1998; ) are all challenges faced by multilingual learners that can be improved through the intentional building of a supportive math community where students feel confident about engaging actively in class and participating in discussions. Activities that promote laughter, teamwork, and engagement, such as those found in *Kickstart: Number Sense*, are known to promote a sense of belonging among students and can help lay a foundation of trust (Pointer et al., 2020).

Student experiences and interactions at school, in the community, and at home shape their identities. Furthermore, building a math community provides a space where students will learn and grow together, letting each student know they are valued, respected, and important (SanGiovanni et al., 2021). This positive atmosphere enhances overall academic performance and well-being. *Kickstart: Number Sense* fosters community building by integrating activities where students are able to share their own mathematical thinking, listen to that of others, and collaborate with peers. In the activity Adding Within Five, students hear Cota Kangaroo say “ $3 + 3 = 5$ .”

Students not only catch Cota’s mistake but also collaborate with each other on how to teach Cota the correct way to solve the problem and demonstrate their own solutions. When students share their approach or their thinking, they establish their own math identity, and peers learn new perspectives.



## Conclusion

Number sense instruction helps young learners understand and work with numbers in a flexible and intuitive way. It goes beyond just memorizing math facts and involves developing a strong foundation in how numbers relate to each other. When students have good number sense, they can visualize numbers, solve problems mentally, and use math in real-life situations.

*Kickstart: Number Sense* continually draws on key research to design and iterate the program as a meaningful intervention for early math education. The program focuses on key numerical skills, utilizes effective teaching strategies, offers developmental progression from concrete to representational to abstract learning, and engages students through various multimodal activities. Through targeted, intentional, and engaging instruction, *Kickstart: Number Sense* lays the foundation of key skills that are necessary for students to access the K–2 math curriculum. Essential foundational skills including counting, number recognition, basic operations, spatial awareness, and language are the core focus of the program to help foster number sense in students. The program is structured to cater to the diverse learning needs of students and provides a scaffolded approach to strengthen comprehension and mastery of mathematical concepts.

*Kickstart: Number Sense* engages students in learning by incorporating a multisensory approach. The program includes hands-on, interactive activities with manipulatives, songs, and games, promoting cognitive function and positive outcomes. Partner talk fosters community and language development, while motivational supports encourage autonomy and competence. The program’s explicit, systematic approach aims to create an enjoyable and supportive learning environment, enhancing students’ number sense development.

*Kickstart: Number Sense* uses interleaved and spaced practice to deepen learning. The program intentionally targets specific numerical skills critical for future math development, interleaving practice across various skills. The approach challenges students to actively retrieve and apply knowledge, reinforcing connections between concepts and enhancing long-term retention.

*Kickstart: Number Sense* helps students quickly advance their proficiency to grade level by aligning with a developmental progression of instruction that moves from concrete to representational to abstract. The program emphasizes connections among different mathematical representations, fostering comprehension and problem-solving. It acknowledges diverse learning needs and provides a scaffolded approach, ensuring students build a strong foundation. The program's assessment tools, including diagnostic assessment and progress monitoring tools, enable instructors to tailor instruction based on individual needs, track student understanding, and adapt teaching strategies for effective learning.

*Kickstart: Number Sense* strengthens students' math identity by promoting ownership of learning, instilling confidence, and fostering active engagement in a math community. The program encourages students to reflect on their unique math journeys, and by leveraging the character Cota Kangaroo, it makes math relevant and relatable. The program establishes a positive and inclusive learning environment through a small-group setting that cultivates a sense of belonging, particularly crucial for underrepresented students.

With *Kickstart: Number Sense*, young learners are equipped with a strong foundation in mathematics, laying the groundwork for success in math for years to come. This program prioritizes short-term outcomes by fostering comprehensive mathematical development in students through multimodal activities. Short-term outcomes include enhanced reasoning skills, deepened understanding of numbers, and a strong number sense for accelerated readiness for the grade-level core curriculum with increased confidence and improved retention. Short-term benefits also include effective communication of mathematical concepts and support for teachers through tailored interventions, differentiated instructions, and progress monitoring tools.

In the long term, *Kickstart: Number Sense* aims to cultivate enduring changes in students' attitudes and approaches to mathematics beyond immediate skill development. The program seeks to build a lasting sense of self-efficacy, confidence, and independence in mathematical skill development. Additionally, it aims to narrow the number sense knowledge gap, promote a sense of community, and foster students' strong ownership of their mathematical skills over time, empowering them to tackle increasingly challenging problems and encouraging curiosity through relevant and interesting contexts.

# KICKSTART Number Sense

---

## Ready to Improve Your Students' Number Sense?

---

Use these questions to reflect on your current practice, considering the research featured in this paper.

- How do you currently provide math intervention to your struggling students?
- Are your students engaged in interactive and motivating number sense activities?
- Do your lessons provide an efficient way to help students access grade-level math?
- How do you monitor student progress in number sense skills?
- Are your students developing more automaticity in their math skills?
- Are your students provided with clear feedback and number sense support?
- Do your students have opportunities to practice the math skills across various contexts?



## REFERENCES

- Agarwal, P. K., & Agostinelli, A. (2020). Interleaving in Math: A Research-Based Strategy to Boost Learning. *American Educator*, 44(1), 24.
- Agarwal, P. K., & Bain, P. M. (2019). *Powerful teaching: Unleash the science of learning*. Jossey-Bass.
- Aguirre, J., Mayfield-Ingram, K., & Martin, D. (2013). The impact of identity in K–8 mathematics: Rethinking equity-based practices. The National Council of Teachers of Mathematics.
- Aunio, P. & Räsänen, P. (2016). Core numerical skills for learning mathematics in children aged five to eight years – a working model for educators, *European Early Childhood Education Research Journal*, 24:5, 684–704, DOI: 10.1080/1350293X.2014.996424
- Beck, M., & Malley, J. (1998). A pedagogy of belonging. *Reclaiming children and youth*, 7(3), 133–137.
- Boaler, J. (2017). *Seeing as Understanding: The Importance of Visual Mathematics for our Brain and Learning*. YouCubed at Stanford University.  
Retrieved from <https://www.youcubed.org/wp-content/uploads/2017/03/Visual-Math-Paper-vF.pdf>
- Bonny, J. W., & Lourenco, S. F. (2013). The approximate number system and its relation to early math achievement: Evidence from the preschool years. *Journal of Experimental Child Psychology*, 114(3), 375–388.
- Brookhart, S. (2017). *How to give effective feedback to your students* (2nd ed.). ASCD.
- Brown, P. C., Roediger III, H. L., & McDaniel, M. A. (2014). *Make it stick: The science of successful learning*. Harvard University Press.
- Bruner, J. S. (1964). The course of cognitive growth. *American Psychologist*, 19(1), 1–15.  
<https://doi.org/10.1037/h0044160>
- Bruner, J. S. (1966). *Toward a theory of instruction*. Cambridge, MA: Belkapp Press.
- Bruner, J. (2004). A short history of psychological theories of learning. *Daedalus*, 133(1), 13–20.
- Carpenter, T. P., Fennema, E., Franke, M.L., Levi, L., & Empson, S. B. (2015). *Children’s Mathematics*. Heinemann Educational Books.
- Clarke, B., & Shinn, M. R. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School psychology review*, 33(2), 234–248.
- Curriculum Associates (2023). *State of Student Learning in 2023*.  
<https://www.curriculumassociates.com/research-and-efficacy/annual-report-the-state-of-student-learning-in-2023>
- Finley, A. M. (2018). Fostering success: Belongingness pedagogy for English language learners. *BC TEAL Journal*, 3(1), 37–48.

## REFERENCES

- Frye, D., Baroody, A. J., Burchinal, M., Carver, S. M., Jordan, N. C., & McDowell, J. (2013). Teaching math to young children: A practice guide (NCEE 2014-4005). Washington, DC: National Center for Education Evaluation and Regional Assistance (NCEE), Institute of Education Sciences, U.S. Department of Education. Retrieved from the NCEE website: <http://whatworks.ed.gov>
- Garon-Carrier, G., Boivin, M., Guay, F., Kovas, Y., Dionne, G., Lemelin, et al. (2016). Intrinsic motivation and achievement in mathematics in elementary school: A longitudinal investigation of their association. *Child development*, 87(1), 165–175.
- Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of Experimental Child Psychology*, 77(3), 236–263.
- Gersten, R., Chard, D. (1999). Number Sense: Rethinking Arithmetic Instruction for Students with Mathematical Disabilities. *The Journal of Special Education* 33.1: 18–28.
- Google Trends. (2015). Google Trends. <https://trends.google.com/trends/explore?date=today%205-y&geo=US&q=math%20intervention,reading%20intervention&hl=en-US>
- Hattie, J., Fisher, D., Frey, N., Gojak, L. M., Moore, S. D., & Mellman, W. (2016). Visible learning for mathematics, grades K-12: What works best to optimize student learning. Corwin Press.
- Hauser, J. (2009). Concrete-Representational-Abstract Instructional Approach. Retrieved from the Access Center: Improving Outcomes for All Students K – 8.
- Howden, H. (1989). Teaching Number Sense. *The Arithmetic Instructor*, 36(6), 6–11. <https://doi.org/10.5951/at.36.6.0006>
- Hyde, D. C., Khanum, S., & Spelke, E. S. (2014). Brief non-symbolic, approximate number practice enhances subsequent exact symbolic arithmetic in children. *Cognition*, 131(1), 92–107.
- Jäncke, L. Music, memory and emotion. *J Biol* 7, 21 (2008). <https://doi.org/10.1186/jbiol82>
- Jordan, N. C., Glutting, J. J., Dyson, N., Hassinger-Das, B., & Irwin, C. M. (2012). Building kindergartners' number sense: A randomized controlled study. *Journal of Educational Psychology*, 104(3), 647–660. <https://doi.org/10.1037/a0029018>
- Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and individual differences*, 20(2), 82–88.
- Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development*, 74(3), 834–850.

## REFERENCES

- Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). Arithmetic fact mastery in young children: A longitudinal investigation. *Journal of Experimental Child Psychology*, 85(2), 103–119.
- Jordan, N. C., Kaplan, D., Locuniak, M. N., & Ramineni, C. (2007). Predicting first-grade math achievement from developmental number sense trajectories. *Learning disabilities research & practice*, 22(1), 36–46.
- Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, 45(3), 850.
- Karpicke, J. D., & Roediger III, H. L. (2007). Expanding retrieval practice promotes short-term retention, but equally spaced retrieval enhances long-term retention. *Journal of experimental psychology: learning, memory, and cognition*, 33(4), 704.
- Lauricella, A. R., Gola, A. A. H., & Calvert, S. L. (2011). Toddlers' learning from socially meaningful video characters. *Media Psychology*, 14(2), 216–232.
- Madan, C. R., & Singhal, A. (2012). Using actions to enhance memory: effects of enactment, gestures, and exercise on human memory. *Frontiers in Psychology*, 3. <https://doi.org/10.3389/fpsyg.2012.00507>
- McIntosh, A., Reys, B. J., & Reys, R. E. (2005). A proposed framework for examining basic number sense. In *Subject Learning in the Primary Curriculum* (pp. 209-221). Routledge.
- Miller-Cotto, D., & Lewis Jr, N. (2020). Am I a “math person”? How classroom cultures shape math identity among Black and Latinx students. <https://osf.io/hcqst/download>
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*.
- Neergaard, L. (March 26, 2013). *Early Number Sense Plays Role in Later Math Skills*. Associated Press. Yahoo! News.
- Piaget, J. (1952). *The child's concept of number*. New York: Humanities Press.
- Pointer, L., McGoey, K., & Farrar, H. (2020). *The little book of restorative teaching tools: Games, activities, and simulations for understanding restorative justice practices*. Simon and Schuster.
- Ramirez, G., Gunderson, E., Levine, S., and Beilock, S. (2013). Math Anxiety, Working Memory and Math Achievement in Early Elementary School. *Journal of Cognition and Development*. 14 (2): 187–202.
- Ramirez, G., Hooper, S. Y., Kersting, N. B., Ferguson, R., & Yeager, D. (2018). Teacher Math Anxiety Relates to Adolescent Students' Math Achievement. *AERA Open*, 4(1). <https://doi.org/10.1177/2332858418756052>

## REFERENCES

- Roediger III, H. L., Agarwal, P. K., McDaniel, M. A., & McDermott, K. B. (2011). Test-enhanced learning in the classroom: long-term improvements from quizzing. *Journal of experimental psychology: applied*, 17(4), 382.
- SanGiovanni, J. J., Katt, S., Knighten, L. D., & Rivera, G. (2021). *Answers to Your Biggest Questions About Teaching Elementary Math: Five to Thrive [series]*. Corwin Press.
- Shumway, J. F., & West, L. (2011). *Number sense routines: Building numerical literacy every day in grades K–3*. Stenhouse.
- Skinner, E., Furrer, C., Marchand, G., & Kindermann, T. (2008). Engagement and disaffection in the classroom: Part of a larger motivational dynamic? *Journal of Educational Psychology*, 100(4), 765.
- Snyder, K., Dinkel, D., Schaffer, C., Hiveley, S., & Colpitts, A. (2017). Purposeful Movement: The Integration of Physical Activity into a Mathematics Unit. *International Journal of Research in Education and Science*, 3(1), 75–87.
- Sousa, D. (2015). *How the Brain Learns Mathematics*. 2nd Edition. Thousand Oaks, California; Corwin.
- Sousa, D. A. (2008). *Facilitator's Guide, How the Brain Learns Mathematics*. Corwin Press.
- Sriram, R. (2020, April 13). *The Neuroscience Behind Productive Struggle*. Edutopia.  
<https://www.edutopia.org/article/neuroscience-behind-productive-struggle/>
- Sterner, G., Wolff, U., & Helenius, O. (2019). Reasoning about Representations: Effects of an Early Math Intervention. *Scandinavian Journal of Educational Research*, 64(5), 1–19.  
<https://doi.org/10.1080/00313831.2019.1600579>
- vanMarle, K. (2015). Foundations of the formal number concept: How preverbal mechanisms contribute to the development of cardinal knowledge. In D. C. Geary, D. B. Berch, & K. M. Koepke (Eds.), *Evolutionary origins and early development of number processing* (pp. 175–199). Elsevier Academic Press.  
<https://doi.org/10.1016/B978-0-12-420133-0.00007-7>
- Wang, A. H., Firmender, J. M., Power, J. R., & Byrnes, J. P. (2016). Understanding the program effectiveness of early mathematics interventions for prekindergarten and kindergarten environments: A meta-analytic review. *Early Education and Development*, 27(5), 692–713.