

KICKSTART

Fractions



Foundational Research Paper

RESEARCH-BASED DESIGN AND LOGIC MODEL

Kickstart: Fractions is a targeted small-group intervention designed to deepen students' conceptual understanding and procedural skills with fractions while bolstering grade-level proficiency and fostering positive mindsets about mathematics to build community, curiosity, and confidence in learning.



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Zaner-Bloser, Inc.
800.421.3018
zaner-bloser.com

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How Can This Foundational Research Paper Be Used?

This paper aims to describe the innovative strategies, pedagogical approach, and theoretical and empirical evidence that informs the *Kickstart: Fractions* design. *Kickstart: Fractions* is a targeted small-group intervention developed for students in grades 3 and above who need additional support in building a strong foundation in fraction concepts. While some students may encounter fractions before grade 3, this program focuses on deepening understanding and mastery of key concepts to prepare students for success in fraction and decimal operations, proportional reasoning, and advanced algebraic thinking in middle school and beyond. Each section details how *Kickstart: Fractions* enhances students' foundational mathematical thinking by teaching fractions with context and manipulatives to ensure that students perceive fractions as a proportional relationship. The approach bolsters students' proficiency in grade-level skills and builds students' positive associations with math, ensuring that critical learning gaps are addressed and that students can become curious, confident, active participants in their math learning community.

Key Points

- Students who struggle with fraction understanding in third grade often have key gaps in their learning and need explicit, systematic teaching to acquire tools and strategies to fill those gaps.
- When critical learning gaps aren't filled, students layer new learning onto weak foundations, contributing to ongoing struggles and poor math self-perceptions with steadily advancing grade-level curricula.
- Students who enter third grade without a firm grasp of number sense are more likely to approach fractions with memorization, rather than flexibility using part-whole reasoning and magnitude thinking.
- Students develop a strong understanding of fractions by engaging in activities that involve them physically, mentally, and emotionally, using manipulatives in meaningful, real-life contexts. These experiences help students see fractions as proportional relationships, connecting them to their existing knowledge of numbers and number relationships, rather than viewing them as a separate, unfamiliar system.
- Embedding a motivational structure into lessons increases overall engagement, curiosity, and interest, and helps replace negative associations about math learning, especially for students who have experienced repeated difficulty.
- A carefully sequenced, systematic fraction intervention with intentionally designed practice strengthens learning and retention by ensuring that students revisit topics over time in differing ways.
- Intervention that scaffolds student learning and progresses from concrete to representational to abstract meets students where they are and ensures connections between new and known information for stronger retention.
- Giving students opportunities to discuss math concepts with partners and small groups provides them with ownership over their learning and practice with academic vocabulary.
- Integrated placement checkpoints enable teachers to monitor progress and align content to students' needs, supporting the delivery of personalized math interventions.
- Intervention materials with thoughtfully scripted and carefully paced lessons ensure successful implementation within existing systems where time and staff resources may be limited.

Logic Model for *Kickstart: Fractions*

Problem Statement

Due to disrupted learning and instructional gaps exacerbated by the pandemic, many students in grades 3–5 lack critical foundational skills and confidence in fractions, part-whole relationships, and proportional reasoning. Core math curricula alone do not adequately address these gaps, and schools often lack the resources, staffing, or structured intervention programs necessary to provide targeted support. Without this foundational knowledge and fluency, students struggle to engage with more advanced mathematical concepts, leading to persistent difficulty in math achievement, diminished academic confidence, and negative self-perceptions as math learners.

Purpose

Kickstart: Fractions is a targeted, engaging small-group intervention product that deepens students' conceptual understanding and procedural skills with fractions. The program is designed for easy and effective implementation, making it accessible for instructors at any comfort level with teaching fractions. It bolsters grade-level proficiency, promotes positive mathematical mindsets, and nurtures a classroom culture of community, curiosity, and confidence in mathematics.

Resources

What resources are or could be available?

- Dedicated time for small-group intervention
- A provided model for responsive instruction placement and intervention
- *Kickstart* student-facing learning content and materials for building fraction understanding (reproducible resources, activity cards, kit of hands-on manipulatives)
- *Kickstart* instructor guides with explicit teaching support so that any adult can lead an intervention (three books targeting fundamentals and fraction operations)
- Online portal with digital access to instructor guides and student-facing instructional videos and resources
- Placement Checkpoints that can be readministered to provide pre- and post-test data for each stage.
- Daily progress monitoring sheets which offer a clear view of each student's growth throughout the program.

Strategies and Activities

What will the activities, events, and such be?

- Structured, sequential fraction intervention in the form of 20-minute lessons
- Explicit, systematic instruction targeting foundational fraction skills and operations essential for grade-level math success
- Developmental learning progression that moves from concrete to representational to abstract, meeting students where they are and accelerating learning
- Math discussions with partners and small groups to build ownership and academic vocabulary
- Multimodal and game-based learning experiences provide variety and active engagement
- Math stories integrate relevant, real-life math applications and model perseverance, resilience, and collaboration
- Skills build on prior knowledge and targets priority grade-level standards
- Practice questions and immediate feedback to aid in learning
- Regular formative assessments and integrated placement checkpoints to monitor student progress and personalize instruction

Outputs

What are the initial products of these activities?

- Instructors present more collaborative lessons
- Instructors use placement checkpoints and progress monitoring opportunities to make data-informed decisions
- Students engage actively with math learning
- Students receive immediate feedback during learning, enabling the instructor to offer more differentiation
- Students learn strategies and tools to support fraction understanding, engaging in fraction operations, and ownership in math learning
- Students recognize part-whole and proportional relationships
- Students talk about fractions more frequently and use fraction-related math vocabulary accurately and confidently

Short-Term and Intermediate Outcomes

- Instructors build capacity to facilitate targeted fraction interventions effectively.
- Students improve their understanding of part-whole and proportional relationships, fraction ordering and comparison, and how fractions interact with each other in fraction operations.
- Students retain and apply math learning in novel situations.
- Students are less hesitant to complete math tasks positively impacting their math confidence and attitudes.

Long-Term Outcomes and Impacts

- Students consistently demonstrate grade-level proficiency and reduced need for intensive math intervention in upper grades, positively impacting overall math performance and academic trajectories.
- Students build confidence in their ability to succeed in grade-level math, fostering self-efficacy and positive perception of their math learning potential so they can develop their independence within math skill development.
- Students maintain sustained positive math mindsets, characterized by ongoing curiosity, resilience, and confidence in approaching increasingly complex or novel math content.
- Schools foster collaborative, engaged math learning communities, enhancing overall academic achievement and attitudes.

Assumptions

- Instructors engage in relevant product training.
- Instructors are provided with all the required materials (i.e., 1 set of print materials and 1 kit per instructor) for effective implementation.
- Lessons are implemented fully and consistently.
- Administrators allow various support personnel to engage in tiered levels of support, not just classroom instructors.

What Is Fraction Understanding and Why Is It Important?

WHAT IS FRACTION UNDERSTANDING?

A fraction is commonly defined as a mathematical expression representing a numerical quantity as a relationship between part and whole or a ratio between two quantities. As outlined by Bruce et al. (2022), researchers generally describe five common contexts in which fractions are used (also described as “meanings of fractions”). These include part–whole relationships, part–part relationships (often referred to as ratios), measurement (distance from zero), as a quotient, and as an operator (e.g., “finding one-half of three”). **Fraction understanding refers to the conceptual knowledge of fractions and the ability to apply that knowledge to fraction operations.** In this way, it involves recognizing fractions, understanding how they are represented, and learning how to compare, add, subtract, multiply, and divide them. This knowledge encompasses both performing operations with fractions and, importantly, a deep understanding of what fractions represent in relation to whole numbers and how they are used in different contexts. Researchers emphasize that gaining knowledge about fractions is an essential step in children’s development of numerical skills and that learning about fractions is a child’s first significant opportunity to understand that certain key properties of whole numbers do not apply universally to all numbers (Siegler et al., 2011). Without conceptual understanding, students are left to rely on rote learning of algorithms. Memorizing the “how” unconnected to the “why” becomes difficult to retain and is ineffective in the long term.

Research has explored **underlying reasoning skills that support the development of fraction understanding.** In a summary of this literature, Bruce et al. (2022) emphasize the importance of *proportional reasoning*, or one’s ability to compare quantities, sizes, and amounts; *spatial reasoning*, or the ability to make sense of space; and *magnitude thinking*, or the ability to think about the size or scale of something when making comparisons. For students to understand the quantity of a fraction, they need to appreciate multiple aspects at once, and proportional reasoning allows them to see how fraction quantities relate to each other. Additionally, as students strengthen spatial reasoning skills, they can better grasp concepts of fractions such as partitioning, comparing, and relating parts to a whole.

Strong spatial reasoning provides a visual, concrete way to see and manipulate fractional relationships. Further, magnitude knowledge supports students' understanding of fraction size, comparison, and equivalence. As students strengthen their sense of magnitude, they are better equipped to reason about fractions. The importance of magnitude knowledge is underscored in studies that have shown causal connections between this reasoning and mathematical knowledge and proficiency with both fractions and whole number arithmetic (e.g., Booth & Siegler, 2008; Siegler & Lortie-Forgues, 2015). Rigorous experimental research has demonstrated the power of interventions that emphasize fraction magnitude understanding (see Fazio et al., 2016 and Fuchs et al., 2017).

IMPORTANCE OF FRACTION LEARNING

Math achievement in the elementary and middle grades is an important factor in students' later education and vocational attainment. As technology continues to advance, math skills are becoming increasingly important. Success in math learning has been linked to greater access to college and career options as well as increased income earning potential and better financial decision making (Crawford & Cribb, 2013; Parsons & Bynner, 1997). **Fractions are widely recognized as a fundamental math concept** that serve as an important bridge to more advanced math learning and long-term achievement. Research has shown that a solid grasp of fractions is critical for success in higher-level mathematics (Bailey et al., 2012; Booth & Newton, 2012; NMAP, 2008; Siegler et al., 2012).

Developing a strong understanding of fractions provides the essential foundation for proficiency in key areas of mathematics, such as measurement, probability, and algebra, which are critical for tackling more advanced math concepts (Bruce, et al. 2022). For example, Brown & Quinn (2007) highlighted the critical role that fraction knowledge plays as a prerequisite for algebra. Their research revealed strong connections between students' proficiency in fraction operations and their test scores in algebra.

Beyond mathematics achievement outcomes, **proficiency with fractions is also important for academic achievement in the sciences, and performance in a wide range of occupations.** It's estimated that two thirds of the US workforce use fractions daily in their jobs (Handel, 2016). Further, knowledge of fractions is immediately applicable to and important for numerous everyday tasks (e.g., cooking, financial tasks, time management, travel and navigation, health, sports, etc.).

WHY IS FRACTION UNDERSTANDING DIFFICULT FOR STUDENTS?

The National Mathematics Advisory Panel highlighted children’s difficulty learning fractions as pervasive, and a barrier to further mathematical development (2008). Curriculum standards across states prioritize fraction instruction in intermediate elementary and early middle grades given the critical role fractions play in building a foundation for more advanced math. To access and approach middle and high school math curricula with a strong foundation for success, it is important for students to become fluent in their ability to apply fraction knowledge to fraction arithmetic and word problems. However, **fraction learning is difficult for many students and adults, a concern in math education that has persisted for many years** (Lortie-Forgues et al., 2015). Studies have shown that even prospective educators and practicing elementary teachers have difficulty with fraction concepts (Copur-Gencturk, 2021; Reeder & Utley, 2017). As a result, teaching fractions can be difficult for teachers who have gaps in their own understanding of the concepts. Additionally, students at risk for math learning difficulties are more likely to face challenges in developing a strong understanding of basic fraction concepts and difficulty closing math learning gaps (Calhoun et al., 2007). Like challenges with early numerical skills, difficulty mastering fraction concepts can contribute to or exacerbate existing math anxiety in learners (Dowker et al., 2016).

“A high percentage of U.S. students lack conceptual understanding of fractions, even after studying fractions for several years; this, in turn, limits students’ ability to solve problems with fractions and to learn and apply computational procedures involving fractions.”

Siegler et al. (2010)

Mastering fractions is challenging due to their complexity, requiring both concrete and abstract thinking, as well as a strong grasp of the underlying concepts and procedures for applying them (Bruce et al., 2022). Researchers have aimed to identify sources of difficulty with fraction learning. One source of challenge is the written notation of fractions, with digits organized above and below a dividing line. Students often have difficulty learning that fractions are numbers that represent quantities. They are susceptible to viewing fractions as arbitrary symbols that are manipulated in unpredictable ways to produce answers a teacher is looking for rather than understanding them as numbers with specific values (Siegler et al., 2010).

Additionally, students often apply their understanding of whole numbers to fractions (referred to as “whole number bias”), which can lead to misconceptions because whole numbers and fractions are different (Gabriel et al., 2013b; Namkung & Fuchs, 2019). Namkung & Fuchs explain three distinctions between whole numbers and fractions that give rise to common errors. First, students are often susceptible to fraction calculation errors when they treat numerators and denominators as separate whole numbers rather than viewing a fraction as a single value. Second, students can use counting strategies when comparing whole numbers, but those strategies aren’t productive when working with fractions, contributing to a tendency to mistakenly apply whole number rules when comparing fractions. Third, students are susceptible to errors when they apply whole number operation procedures; adding and subtracting fractions require a common denominator, while multiplying and dividing fractions do not.

Although overgeneralizing properties of whole numbers to fraction understanding contributes to the challenge inherent in learning fraction concepts, researchers generally agree that a solid understanding of whole number magnitudes leads to better outcomes in fraction learning. As such, **students develop their understanding of fractions as they gradually expand and refine their overall knowledge of numbers**, learning that all numbers have values that can be placed on a number line (see integrated theory of numerical development, Siegler et al., 2011).

In addition to overgeneralization of whole number knowledge to fractions, Bruce et al. (2022) suggest other potential contributors to the challenge of learning fractions. They emphasize that a premature focus on algorithms without full understanding of the meaning of the procedures creates a shaky foundation and leaves students susceptible to confusion between various algorithms they are learning and when to use them. Additionally, they suggest that relying too heavily on certain visual representations, such as circles as the primary and predominant way students are introduced to the idea of dividing something into equal parts can be problematic and students tend to partition circles unevenly when dividing them into more than two parts.

THE IMPORTANCE OF WELL-DESIGNED FRACTION INTERVENTION

Longitudinal research has shown that **students with limited growth in their fraction knowledge during later elementary grades were much less likely to meet state math standards at the end of sixth grade** (Jordan et al., 2017). However, math interventions focused on fractions (Stevens et al., 2018) and more specifically, targeted

interventions aimed at building students' sense of fractions and strong conceptual understanding of fractions as numbers with values and as measurements of quantity have been shown to be beneficial (Dyson et al., 2020; Jordan et al., 2017; Namkung & Fuchs, 2019; Siegler et al., 2010). **Timely intervention is critical to support access to core math curricula** and achievement expectations in the intermediate grades and to set up a strong foundation for later success. A solid grasp of fractions can help ensure that all students have the necessary skills to succeed in mathematics, reducing gaps in understanding that can lead to further difficulties in school. This is especially critical given the troubling decline in US math scores, especially among historically underserved groups, widening achievement gaps between white students and students of color, and negative impact of the COVID-19 pandemic on math proficiency (Hengtgen & Biaggi, 2023).

“**Fourth-grade students who are at risk for failure with the advancing mathematics curriculum, due to histories of poor mathematics achievement in the primary grades, can succeed with challenging mathematics content, if they are provided with a well-designed intervention.**”

Fuchs et al. (2017)

CHARACTERISTICS OF EFFECTIVE FRACTION INTERVENTION

Studies examining the impact of fraction intervention have demonstrated the importance of instruction that is systematic, builds on students' existing understanding, and **intentionally supports the development of key types of reasoning that underlie fraction understanding** (Fazio et al., 2016; Fuchs et al., 2017; Moss & Case, 1999). In their findings from collaborative action research with teachers in grades 3–10, Bruce et al. (2022) suggest a focus on unit fractions as a foundation for building fraction understanding, and ample opportunities for students to build and apply their conceptual knowledge through a visual-spatial approach. They suggest a progression of eight core concepts that build on one another in a generally linear but iterative fashion, moving students from a first concept of estimating proportions to the final concepts of adding and subtracting and then multiplying and dividing fractions.

Additionally, to summarize recommendations based on high-quality research and expertise from a panel of research scholars and expert practitioners, the What Works Clearinghouse presents five recommendations for effective fraction instruction (Siegler et al., 2010):

Recommendations and corresponding levels of evidence.

Recommendation	Minimal Evidence	Moderate Evidence	Strong Evidence
1. Build on students' informal understanding of sharing and proportionality to develop initial fraction concepts.	◇		
2. Help students recognize that fractions are numbers and that they expand the number system beyond whole numbers. Use number lines as a central representational tool in teaching this and other fraction concepts from the early grades onward.		◇	
3. Help students understand why procedures for computations with fractions make sense.		◇	
4. Develop students' conceptual understanding of strategies for solving ratio, rate, and proportion problems before exposing them to cross-multiplication as a procedure to use to solve such problems.	◇		
5. Professional development programs should place a high priority on improving teachers' understanding of fractions and of how to teach them.	◇		

Recommendation 1 describes a developmental progression that helps students move from understanding fractions through sharing activities and dividing sets or whole objects to building proportional reasoning and reinforcing foundational concepts of fraction ordering and equivalence. *Recommendation 2* encourages using number lines to help students understand equivalence, fraction density, and various fraction representations. *Recommendation 3* suggests addressing the common misconceptions that students are often susceptible to when operating with fractions. The use of real-world contexts and meaningful models is emphasized to help students connect conceptual understanding with procedural knowledge and make sense of procedures instead of relying on rote memorization without meaning. This dovetails with recent research demonstrating the importance of providing students with opportunities to build understanding of fraction procedures alongside conceptual knowledge development, rather than massing instruction of one before the other (Running et al., 2023). *Recommendation 4* amplifies the importance of ensuring students understand the “why” behind operation procedures and the benefit of providing them with opportunities to use and discuss strategies for problem solving. Finally, *Recommendation 5* highlights the importance of addressing teacher and support staff knowledge gaps in fractions and emphasizes integrating professional learning to ensure high-quality instruction.

The Goals of *Kickstart: Fractions* Instruction

***Kickstart: Fractions* addresses the overarching goals of building students' deep conceptual understanding of what fractions are, how they interact in various operations, and how to visualize and apply fraction math in daily life.** These factors are integral to effective fraction instruction and building students' overall mathematical knowledge foundation (Siegler et al., 2010). Fraction instruction in the elementary grades typically emphasizes fractions as parts of a whole. Research indicates that although understanding fractions as part of a whole is crucial, teaching should also help students recognize that fractions are numbers with distinct values or magnitudes. These values can be compared, ordered, and seen as equivalent to other fractions, which is key to avoiding common misconceptions that may hinder their understanding of fractions (Bruce et al., 2022; Siegler et al., 2010).










With an **emphasis on conceptual understanding**, the *Kickstart: Fractions* approach positions students to understand and apply fraction procedures required in their core math curriculum. It aims to increase students' understanding of fractions as numbers that extend the number system beyond whole numbers, shoring up what students know about whole numbers and expanding their number sense to fraction understanding. Studies have demonstrated that students are more proficient with computational procedures when they know why those procedures make sense (Cowan et al., 2011; Rittle-Johnson, 2017). *Kickstart: Fractions* is designed to build and strengthen students' number sense with fractions, leveraging visual representations and a developmental progression that moves students from concrete to representational to abstract understanding. It also aims to teach students the reasoning behind fraction computation procedures.

Furthermore, the approach helps to **build students' repertoire of strategies for approaching fractions**, and overall math learning more broadly. In this way, it aims to strengthen students' ability to engage in proportional reasoning, a prerequisite for more advanced math learning (National Council of Teachers of Mathematics, 2000; National Mathematics Advisory Panel, 2008). Research has shown that students often struggle, or are not adequately taught, to recognize mathematical similarities across different problems, which makes it challenging to apply known strategies to different contexts (Riccomini et al., 2016; Siegler et al., 2010). As aligned with instructional best practice evidence, the program directly addresses increasing students' flexible problem-solving and application to authentic scenarios that bring relevance to the learning.

Alongside building students' conceptual understanding and strategy use, an essential aim of *Kickstart: Fractions* is to **assuage emerging or existing negative perceptions about math**, and specifically, fraction learning, among both students and instructors. The strong link between math beliefs, perceptions, and achievement is well-established. Research has demonstrated the influence of classroom environments, math learning experiences, and teacher practices on students' math beliefs and performance (Gilbert et al., 2014; Wang, 2012), as well as how students' enjoyment of math impacts their effort, beliefs, and confidence (Christensen & Knezek, 2020; Pinxten et al., 2014; Schukajlow & Rakoczy, 2016).

Additionally, teachers play an essential role in helping students gain proficiency with fractions. Yet many instructors tasked with supporting students experiencing math difficulties may not feel prepared or confident in delivering this targeted instruction (Copur-Gencturk, 2021; Li & Kulm, 2008; Newton, 2008). *Kickstart: Fractions* is intentionally designed to foster student motivation, enjoyable math learning experiences, and a positive math self-concept. It is built with understanding that math anxiety is often present in math intervention students in the intermediate grades, and that teacher attitudes, classroom culture, and assessment practices play a powerful role in shaping the experiences of these learners (see summary in Sousa, 2015). It also aims to support successful implementation by any adult, regardless of prior knowledge and teaching experience, empowering both the instructor and the student. By enhancing the instructor's conceptual understanding of fractions, the program strengthens the critical knowledge base of adults, ensuring more effective interventions and fostering positive teaching and learning experiences.

KICKSTART: FRACTIONS // Full Intervention Kit

<p>Book 1 <i>Foundations</i></p> <p>Covers:</p> <ul style="list-style-type: none"> • Grade 3 Math Standards • Grade 4 Math Standards • Includes Placement Checkpoints 			<p>MANIPULATIVES / RESOURCES:</p> <ul style="list-style-type: none"> • Stage D Fraction Bar / Number Line Cards (3) • Stages E & F Fraction Bar / Number Line Cards (3) • Fraction Bars (7) • Pattern Blocks: hexagons (24), rhombuses (50), trapezoids (48), triangles (80) • Transparent Spinners (4) • Fraction Tiles (7) • Dice (5) • Sheet Protectors (7) • Dry Erase Pens (7) • Transparent Fractions Overlay Sheets (7)
<p>Book 2 <i>Adding and Subtracting</i></p> <p>Covers:</p> <ul style="list-style-type: none"> • Grade 4 Math Standards • Grade 5 Math Standards • Includes Placement Checkpoints 			<p>DIGITAL COMPONENTS:</p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p>Online Portal</p> <ul style="list-style-type: none"> • Digital Access to: <ul style="list-style-type: none"> • Lesson Pages • Teacher Guides • Reproducibles • Activity Cards </div> <div style="text-align: center;">  <p>Instructional Videos</p> <ul style="list-style-type: none"> • 11 Videos • 1-2 minutes, stand-alone, animated segments showcasing fraction concepts for students </div> </div>
<p>Book 3 <i>Multiplying and Dividing</i></p> <p>Covers:</p> <ul style="list-style-type: none"> • Grade 4 Math Standards • Grade 5 Math Standards • Includes Placement Checkpoints 			<p>Teacher Guides</p> <ul style="list-style-type: none"> • Three 9" x 12" Books • Full Color, Spiral Bound
<p>Reproducible Resources</p> <ul style="list-style-type: none"> • Three 8.375" x 10.875" Books • B/W, Spiral Bound 	<p>Activity Cards</p> <ul style="list-style-type: none"> • Three 8.375" x 10.875" Books • Full Color, Spiral Bound 		<p>© 2026</p>

APPROACH AND PROGRAM COMPONENTS

All *Kickstart* math intervention products are designed to support students with gaps in key foundational math skills that hinder their progress toward grade-level proficiency. Created by experienced teachers and built on evidence-based strategies, these products make math intervention effective while fostering positive learning experiences. Students gain conceptual understanding and confidence, enabling them to approach, engage with, and discuss their math learning successfully.

Kickstart: Fractions is designed to deliver targeted math instruction for students typically in grades 3 and above who need intervention in key skills required to understand fractions, helping them succeed in core mathematics instruction and progress toward grade-level proficiency. The program was specifically created to help students develop deep conceptual understanding of fractions, necessary for performing fraction operations accurately and fluently, and build a solid foundation for more advanced math concepts. *Kickstart: Fractions* draws on a multidisciplinary body of scientific research that informs effective math interventions, fraction instruction practices, and positive learning environments. Central to the approach is providing students with ample opportunities to discuss their math ideas with peers, engage in meaningful learning activities, reflect on their learning, experience success with math, and gain ownership over their math learning.

Kickstart: Fractions lessons are designed for small groups and can be taught by any instructor. Each lesson is fully scripted, outlines everything the instructor needs to know, and includes all necessary materials in the classroom kit. Built with intervention groups in mind, the program supports students of different ages and skill levels in groups of up to six, engaging them in instructional activities that bridge specific skills and prepare them for grade-level work. As a flexible intervention, it provides 20-minute small-group instruction that fits seamlessly into various schedules and meets diverse student needs. The ready-to-use kits minimize preparation time and include assessments for student placement, progress monitoring, and instructional guidance.

Kickstart: Fractions is designed to align with student needs, maximizing outcomes and providing an efficient approach to intervention. Students join small groups with peers at similar skill levels to receive targeted instruction that builds toward grade-level proficiency. With six entry points across three books, the program enables instruction tailored to each student's specific needs. Consistent progress monitoring allows instructors to evaluate student learning and make informed instructional decisions as students progress through

the content. The program includes daily formative assessments—exit tickets—to track student understanding of each lesson.

Kickstart: Fractions maintains a targeted focus on essential rational number skills critical for children’s future mathematical development. It is designed to support students with knowledge gaps or those experiencing math difficulties and frustration, helping them build the foundational skills needed to access grade-level core curriculum. When used as intended, the program is expected to produce the following outcomes.

SHORT-TERM OUTCOMES

The short-term outcomes of the *Kickstart: Fractions* math intervention program focus on strengthening students’ ability to master grade-level math skills by building the conceptual reasoning and procedural knowledge required for deep fraction understanding and success with fraction operations.

Through structured lessons, meaningful learning contexts, and multimodal activities, students improve their understanding of part–whole and proportional relationships, the concept of a fraction, and how fractions interact with one another in operations. These activities, grounded in key reasoning skills such as proportional reasoning, spatial reasoning, and magnitude thinking (Bruce et al., 2022), help students build a foundation for future success in math.

Students engage in math discourse, building both language skills and a deeper understanding of mathematical concepts (Sarama et al., 2012; Webb et al., 2021). *Kickstart* lessons bridge prior knowledge with new concepts, advancing students’ thinking from concrete to abstract and connecting math to real-world contexts. As a result, students find math more relevant, strengthen retention, and increase their ability to apply concepts in novel situations.

Research suggests that academic success achieved through effort and strategic thinking boosts students’ self-esteem and motivation (Meltzer et al., 2001). As students build stronger conceptual understanding and experience success—especially those who have struggled with math in the past—they become less hesitant to tackle math tasks, develop confidence in their abilities, and view themselves as capable math learners.

LONG-TERM OUTCOMES

Like all *Kickstart* products, the long-term outcomes of *Kickstart: Fractions* go beyond immediate math skill development, aiming to foster lasting changes in students' attitudes and approaches to mathematics. Research on learner self-confidence underscores the importance of providing opportunities for students to verbalize their self-awareness, receive positive feedback, share knowledge with peers, view themselves as capable, and recognize the positive results of their efforts (Maclellan, 2014).

Through their experiences with *Kickstart: Fractions*, students are expected to build confidence in their ability to succeed in grade-level mathematics, strengthening their self-efficacy and fostering a positive perception of their potential in math. This growth enables them to become more independent in developing their math skills. As their confidence grows, students develop a sense of ownership over their math abilities, empowering them to tackle increasingly challenging problems with resilience.

Additionally, students will make meaningful connections in their learning, engage with relevant and novel contexts, and collaborate and communicate effectively with peers, cultivating curiosity for math and a sense of community. High-quality fraction interventions have been shown to close math achievement gaps for elementary students at risk of poor outcomes (Fuchs et al., 2021b). The program aspires to narrow these gaps in fraction understanding, which often pose barriers to grade-level math achievement, leading to better outcomes for more students in advanced math and reducing the need for costly, less impactful interventions in middle or high school.

How Does *Kickstart: Fractions* Help Students Strengthen Fraction Understanding?

ENGAGE STUDENTS IN LEARNING

Students' engagement in learning is a critical factor linked to achievement

outcomes (Lei et al., 2018; Wang & Eccles, 2012), and research suggests that boosting students' engagement in learning may be a key strategy for enhancing academic performance (Headden & McKay, 2015). Engagement in the earlier grades is predictive of academic attainment in later grades, and over time, students' engagement in and attitudes toward learning are important protective factors that help offset negative outcomes such as dropping out of school (Finn & Zimmer, 2012). Students who demonstrate academic engagement behaviors such as staying attentive, completing homework, being prepared, and actively participating in class activities tend to perform better academically than less engaged students, and this is particularly crucial for students most academically at risk (Finn & Zimmer, 2012).

Additionally, the research demonstrates that engagement impacts students both directly (e.g., paying attention, completing assignments) and indirectly (e.g., behavior that interrupts instruction or hinders learning opportunities), and that the connection between engagement and achievement is reciprocal. In other words, high achievement is likely to foster students' continued engagement while low achievement is likely to hinder it (Finn & Zimmer, 2012). Furthermore, students who demonstrate higher levels of engagement in the classroom tend to receive more teacher involvement, and students who appear disengaged tend to receive teacher responses that further diminish their motivation (Skinner & Belmont, 1993).

Scholars describe student engagement broadly as ongoing motivation for learning and continuous growth, and it is often conceptualized as comprising affective, behavioral, and cognitive engagement (Fung et al., 2018). It is considered multifaceted and malleable, or able to be changed (Fredricks et al., 2004). Student engagement is also described in terms of learning engagement or school engagement, with a recently proposed framework suggesting that students can become disengaged academically, socially, or both (Wong & Liem, 2022). This work emphasizes the importance of considering the degree to which students may be engaged in learning activities and/or feel a sense of connection within their school community.

Specific to learning engagement, academic engagement is a dynamic state that can change based on students' individual characteristics or the nature of the learning activity. In this way, thoughtful design of learning activities and the teacher's approach are important in supporting students' motivation for and interest in learning, and their self-regulation skills, both critical for building learning engagement. **Instruction needs to consider ways to elicit students' affective engagement** (i.e., feelings about learning), **behavioral engagement** (i.e., on-task and participatory behaviors), **and cognitive engagement** (i.e., use of cognitive strategies for learning).

Additionally, research exploring the influence of prior knowledge and demands on working memory capacity during learning finds that students' engagement in learning is strongly influenced by their prior knowledge; students are more engaged when their prior knowledge is higher, and their cognitive load is lessened (Dong et al., 2020).

Studies exploring relationships between student engagement and achievement in mathematics show that all three kinds of engagement (i.e., affective, behavioral, and cognitive) are positively related to math outcomes. This work also underscores the importance of building a **learning environment** that intellectually challenges students, promotes their active participation through interesting and meaningful activities, and fosters their sense of belonging (Fung et al., 2018).

Research also indicates that students bring diverse patterns of **math-related motivational beliefs** to the classroom, and these patterns can differ by gender and race/ethnicity, as well as by their levels of math achievement and behavioral engagement. Importantly, this work suggests that students with lower behavioral engagement in math class may benefit from intervention that promotes positive beliefs about math while simultaneously creating more opportunities for students to experience success and perform better in math (Hsieh et al., 2021).

Furthermore, research highlights the importance of students' **growth orientation** in math as a factor in their engagement. Growth orientation encompasses the degree to which students believe their math capability is malleable and set their own goals for self-improvement and task-based improvement. Not only are better math outcomes associated with growth-oriented students, but collaborative, growth-oriented classroom environments with teachers who demonstrate growth mindsets themselves are also linked to better achievement (Bostwick et al., 2020). Classroom environments where teachers are flexible, adapt to student needs, and foster student's growth orientation in math build learning engagement important for math achievement.

How Does *Kickstart: Fractions* Engage Students in Math Learning?

Kickstart: Fractions lessons contain embedded features designed to motivate and engage students in math learning as well as enable positive teacher attitudes. This approach creates a safe, collaborative intervention culture where students are comfortable taking risks, collaborating with peers, and building a sense of belonging in their intervention group and in a broader community of math learners. Concepts are taught through multimodal activities and experiences that provide students with variety, pique their interest, and foster their active engagement and self-reflection. Activities continually build and activate students' prior knowledge to reduce overwhelm and provide opportunities for students to be meaningfully involved in their learning. Activities also promote group cohesion, collaboration, growth orientation for learning, and opportunities for students to feel successful.

Deep Dive

At the core of *Kickstart: Fractions* are learning activities designed to **boost students' motivation and self-regulation**, fostering and maintaining their engagement in math learning. These embedded features address the multidimensional nature of learning engagement, including students' feelings about and connection to the learning process, active participation in learning tasks, persistence with challenging concepts, self-regulation, and the use of effective strategies. Research underscores the importance of this approach in math intervention, as students are more likely to engage and less likely to resort to avoidance strategies in classrooms that emphasize learning, understanding, effort, and enjoyment. In such environments, teachers play a key role in supporting students to build understanding and demonstrate new skills (Turner et al., 2002).

Kickstart: Fractions creates a **learning context that fosters student engagement** in several ways. Each lesson utilizes a **similar routine**, so students gain familiarity with the flow of the lesson and comfort in knowing what to expect when they participate. Lessons begin with **Warm-Up activities** designed to get students ready to learn. These activities promote students' curiosity and interest as they approach the learning task, while also considering the age and development of students who enter the program.

Students who are identified for fraction intervention may already believe, or may be starting to believe, that math is not for them. Given the close relationship between feelings about learning and learning behaviors (Finn & Zimmer, 2012), students may

arrive at an intervention group with emotions, attitudes, beliefs and perceptions that hinder learning engagement. The lesson Warm-Ups are quick initial activities designed to help move students from an emotional, “fight, flight, or freeze” state into their prefrontal cortex “thinking” state (Arnsten, 2009; Whiting et al., 2021).

Additionally, a non-threatening setting supports cognitive processing (Lyons & Beilock, 2012). To this end, these initial activities are meant to feel non-threatening, easy, and achievable, and are crafted to build student confidence while sending signals to the brain that it doesn’t need to have its guard up and it is safe to learn. Warm-Ups also include specific strategies for building group cohesion to allow students to more willingly share their ideas. For example, in Book 3, Lesson 17, students are asked to reflect on a hypothetical scenario where they are at home and stuck with a project for school. They discuss who they may go to for help in this instance. Warm-Ups like this provide students with space to share their own experiences, which helps encourage others to share and gain new strategies in general, and then more specifically in math.

Some Warm-Ups center specifically on fostering positive beliefs and perceptions about math learning. In one lesson (Book 1, Lesson 15), students are asked to vote on whether they’d rather spend one minute solving an easy math question without recognition from others or five minutes solving a difficult math question and receive praise from someone. They are given time to think and then share with the group. This activity fosters self-reflection on perseverance behavior, a necessary skill in math education.

STAGE D

Lesson 15: Comparing Fractions with the Same Numerator

MATERIALS

Copy in advance:

- Team Coin Tracker (ongoing)
- Lesson 15 CRANE-ium Competition (R25, 1 for the group)

For the teacher:

- Activity Cards 42–47
- Sheet protector
- Dry erase pen

Per student:

- Pencil

WARM-UP

T: Today is a vote. Stand up and get ready to vote with your arms but don’t move until I say “Ready? Vote.” The question today is would you rather spend one minute solving an easy math question and get no recognition or five minutes solving a difficult math question and get praise from someone? Here’s how to vote: arms all the way up if you’d rather do something easy for no one to notice. Arms all the way out to your side if you’d rather solve a difficult question and get praise. Ready? Vote.

Students may be seated after the vote and share their ideas if they’d like. If a student has their arms out, ask them who they would like to have notice their work. This may look like a would-you-rather activity. In fact, stretching their arms signals to their brains that it is safe enough to think and learn.

Daily **partner talk** is another key feature designed to provide a safe culture for students that fosters engagement through a sense of belonging and valuing-learning activities. Regular peer discussion provides students with opportunities to voice their ideas in supportive partnerships, buoying confidence necessary for sharing ideas with the larger group and eventually, more broadly within the whole classroom setting. In fact, research demonstrates how students with limited mathematical knowledge can form new connections between mathematical concepts and representations, while also broadening their problem-solving strategies as a direct result of their engagement (Webb et al., 2021).

Finally, research suggests the use of authentic tasks and includes practical or interactive learning to promote the interest, motivation, and pleasure of students in learning. *Kickstart: Fractions* integrates these types of learning opportunities regularly throughout the lesson activities. For example, in Book 2, Lesson 3, students listen to a story about the character Rory the Flamingo and his walk to work, and are asked to use their fraction bars to model the story (e.g., Rory walked $\frac{3}{10}$ of one mile to get a coffee, then walked $\frac{4}{10}$ of one mile and stopped to talk to a friend.). This is a meaningful application of adding fractions through distance. In this type of activity, students are consistently utilizing their contextual knowledge of a scenario with a concrete model to enhance their understanding.

Kickstart: Fractions Book 2
Lesson 3 Landing Name: _____

After work, Rory started on the 1 mile walk home from the bakery. Rory walked $\frac{5}{10}$ of a mile to get a cheeseburger, then stopped after another $\frac{3}{10}$ of a mile to pet a neighbor's dog. How much farther does Rory have to walk to get home?

$\frac{5}{10} + \frac{3}{10} = \frac{8}{10}$, so $\frac{2}{10}$ of a mile more until Rory gets home.

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
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Today's learning target:
 I can add fractions to make one whole.

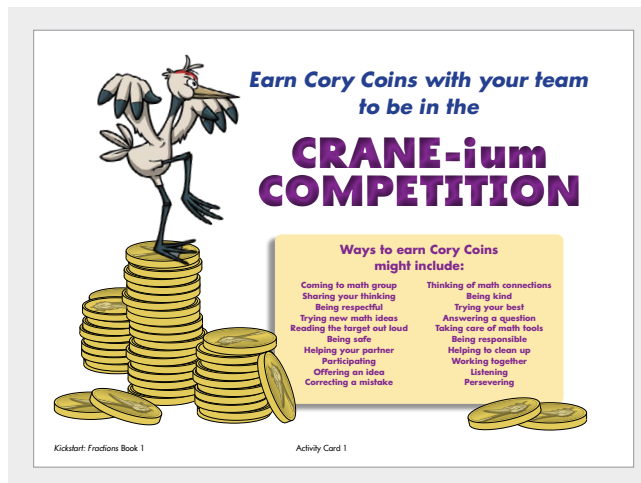
Self-Reflection:
 I can teach this to another student.
 I can do this on my own.
 I can do this with some more practice.

Kickstart: Fractions Book 2 8.7 Lesson 3 Landing

Along with eliciting students' affective engagement, or the emotional aspects of a student's involvement in the learning process, *Kickstart: Fractions* is thoughtfully designed to promote students' cognitive and behavioral engagement. Lessons provide structure through clear guidelines and expectations, ensuring an appropriate level of challenge, and providing students with positive feedback on their growing math capabilities. Research suggests that these strategies help students feel a sense of control and competence, which enables them to put forth more effort and persist longer with learning tasks (Wong & Liem, 2022).

The program's **built-in structure for student motivation** includes gathering team points in the form of Cory Coins during cooperation-based group activities. Students can earn team points daily for actions such as participating, completing work, cooperating with peers, sharing their ideas with the group, recognizing a mistake, or persevering through a difficult task. As part of gathering points, students have the opportunity to roll a die every few lessons. Depending on the outcome of the roll, coins will be either added or subtracted to the team total. The random nature of a dice roll keeps students engaged by releasing dopamine (the neurotransmitter related to reward and motivation). Students will associate excitement with group participation, increasing their desire to come back and a willingness to learn (Willis, 2010).

Additionally, at the end of a set number of lessons, students have the opportunity to compete in a team cooperative activity called the CRANE-ium Competition, where teams choose a task and then work together toward an answer. The coins and activities give instructors opportunities to praise students for active participation, attentive learning behaviors, and positive peer interactions, rather than focusing on praise for quick or correct answers, which is a less productive means for fostering engagement and healthy math identities.



Partner talk activities within lessons also provide ample opportunities for students to voice their math thoughts and develop concepts, fostering cognitive engagement. This approach aligns with research recommended practices to facilitate cognitive engagement such as using open-ended tasks that require students to create multiple representations and solutions, offering opportunities to engage with complex problems, and encouraging students to refine their thinking in response to questions (Wong & Liem, 2022). These practices can stimulate students' curiosity and desire for cognitive engagement and combat passive learning.

Brain researcher David Sousa (2015) points out that emotion is the driver of attention, and attention is required to drive the learning. Therefore, we must attend to the emotion before students are ready to learn.

Kickstart lessons also build students' strategy use and self-reflection, which are critical pathways for increasing engagement. Lessons always include a **Learning Target**, providing students with a clear goal for the lesson. Research highlights the importance of teacher clarity (Hattie et al., 2017). Clearly communicating instructional goals focuses students' attention on the objectives and provides success criteria, helping them understand what to prioritize and what is expected in their learning. Clear instructions also reduce the cognitive load on students' working memory, enabling them to engage more deeply with the material and feel less anxious about absorbing lesson content (Bolkan & Goodboy, 2024; Serki & Bolkan, 2023).

Lesson routines conclude with a **Landing**, a structured opportunity for instructors and students to revisit the day's learning objective. This routine allows teachers to monitor progress while giving students a chance to review the learning target and reflect on their progress toward it. Over time, this practice helps students focus more intentionally on learning targets, bringing them closer to achieving those goals (Hattie et al., 2017).

Additionally, increasing self-reflection and metacognitive knowledge enhances effective math strategy use and problem-solving (Braithwaite & Sprague, 2021; Throndsen, 2011). This focus on self-reflection and strategy empowers students to become more engaged and confident math learners.

DEEPEN CONCEPTUAL UNDERSTANDING

Why Is Strong Conceptual Understanding Essential for Learning Fractions?

Conceptual knowledge of fractions is often referred to as understanding the underlying principles and ideas governing fractions, rather than just the ability to perform fraction calculations (e.g., Gabriel et al., 2013a). Deep conceptual understanding forms an **important foundation for children's development of mathematical procedural knowledge and overall mathematical competence**. Robust conceptual understanding is critical for learning fractions because it enables students to fluently use relationships between parts and wholes, utilize proportional reasoning, recognize different representations, and apply fraction concepts in problem solving and fraction operations.

As students build this strong foundational understanding, they are more able to make connections, reason mathematically, and build on their skills to progress to more advanced math topics (Rittle-Johnson, 2017). The What Works Clearinghouse's effective fraction instruction practice guide recommends building students'

conceptual knowledge, especially their understanding of fractions as numbers, what they represent, and why fraction operations work the way they do (Siegler et al., 2010). Research shows that this deep conceptual understanding helps learners recognize that properties of whole numbers don't always apply to fractions, and that numbers, including fractions, have magnitudes that can be ordered in a meaningful way. This understanding is central to children's difficulties with fraction arithmetic (Lortie-Forgues et al., 2015).

Studies have confirmed that conceptual knowledge is a strong predictor of students' fraction procedure knowledge, surpassing other important factors like working memory and whole number knowledge (Siegler et al., 2013) and suggest that learners who leverage conceptual knowledge may have an advantage over those who depend more heavily on procedural knowledge as they are developing math skills (Hallet et al., 2010). Furthermore, when students understand the principles and reasoning underlying fraction procedures, they are better equipped to effectively check their solutions and may be less susceptible to producing implausible answers or confusing procedures as they advance in the math content across the grades (Braithwaite et al., 2017). Without strong conceptual understanding, students are left to build on incomplete knowledge.

Many students identified for math intervention in the elementary grades lack the deep conceptual knowledge required to master fraction concepts and develop the critical foundation for later math learning (NMAP, 2008). This is further compounded by the fact that adults, including teachers, often struggle with fractions, leading to instruction that emphasizes procedure over concept and lacks the depth students need for proficiency (Gabriel et al., 2013a; Perry, 2023). However, interventions that prioritize both conceptual understanding and procedural fluency help students achieve better outcomes and a deeper appreciation for the connection between fractions in math and everyday life.

Fraction intervention studies with elementary learners show improved fraction learning outcomes for programs targeting conceptual understanding (Roesslein & Coddling, 2019). Enhancing students' conceptual understanding of fractions promotes their ability to approach problems from multiple perspectives and apply their knowledge flexibly (Rittle-Johnson, 2017). Students build critical thinking and reasoning skills about fractions that can promote more effective strategy use.

When students understand how concepts connect to procedures, they can more easily retain fraction concepts and transfer this knowledge to new mathematical situations. Students' fraction sense is malleable and responsive to targeted instruction, making it a key intervention priority (Jordan et al., 2017).

How Does *Kickstart: Fractions* Deepen Students' Conceptual Understanding of Fractions?

Kickstart: Fractions leverages evidence-based instructional components, including concrete and visual representations, explicit, systematic instruction, a range of examples, and authentic context to engage students in meaningful practice to enhance deep conceptual understanding of fractions. The program integrates research findings that underscore the importance of teaching students to build fraction sense, proportional and part-whole reasoning, and understanding of fraction magnitudes, necessary for acquiring abstract understanding to make sense of fractions and how they work in fraction operations.

SKILLS PRACTICE

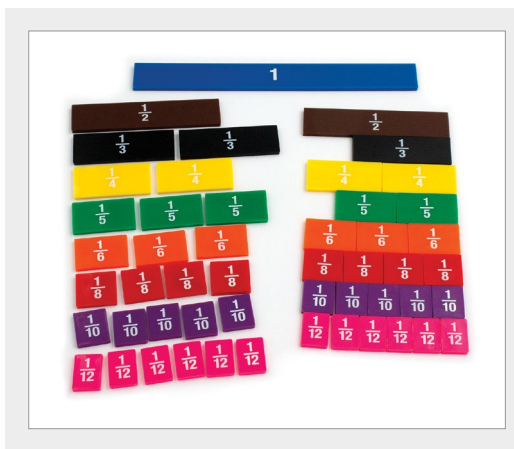
***Kickstart: Fractions* lessons move rapidly through ideas that are foundational to conceptual understanding. Sometimes students require more time to practice each skill. Taking an extra day to practice is a great way to cement students' learning. Optional skills practice for Lessons 8–12 is available as R84 and answers can be found on page 170 at the end of this book.**

Deep Dive

The instructional approach of *Kickstart: Fractions* integrates a variety of strategies for enhancing students' conceptual understanding in fraction learning. Research suggests that to build deep conceptual understanding of fractions, it's critical to address whole number bias and to foster students' ability to understand and represent fraction magnitudes (e.g., Siegler et al., 2010). Fractions require spatial reasoning, and as such, having access to accurate visual representations can help students build their understanding and aid in their ability to compare fractions, establish equivalencies, and complete fraction operations (Bruce et al., 2022).

Kickstart: Fractions instruction emphasizes fractions as numbers and uses varied visuals and hands-on manipulatives to deepen conceptual knowledge. Lessons

utilize fraction bars, pattern blocks, and number lines as essential tools to help students make sense of fractions on a conceptual level. Fraction bars provide concrete, manipulative representations of fraction concepts. Studies of intervention approaches show that using manipulatives in math instruction positively benefits student learning, especially when manipulatives are not perceptually distracting and the teacher provides guidance (Carbonneau et al., 2013). When working with fraction bars, students begin with the blank side of the bars (number side down), which allows them to see relationships without being distracted by fractional values.



Fraction number lines are used frequently to provide another visual, concrete way for students to see and understand the relative size and position of fractions. Research underscores the importance of utilizing number lines as a key representational tool to clarify the relationship between fractions and whole numbers for building students' deep understanding of fractions as numbers and units of measure (Barbieri et al., 2020; Roesslein & Coddling, 2019). Number lines help show that numerators and denominators work together to determine the value of the fraction. These linear representations are an important tool to help students visualize relationships between numbers.

Students need these models because part-whole models such as shading parts of a circle aren't sufficient tools to build strong fraction knowledge. For instance, when comparing the size of fractions, *Kickstart* Book 2, Lesson 12 instructs students to first compare with physical items such as fraction bars to have them tactilely engage with the proportion of the part to the whole. Students are then able to transfer that understanding to more representational models like number lines and drawings to compare the relative size of fractions in Lesson 13.

In addition to the benefit of varied visuals and concrete manipulatives, **meaningful applications** are important to support students' conceptual understanding and transfer of learning. Research on the components of effective fraction instruction also finds that applying real-world context to teach math problem solving, along with promoting student engagement, is important for strengthening students'

conceptual understanding. Specifically, anchored instruction that situates learning in a problem-based context and is combined with explicit instruction and a wide range of examples, has been shown to be beneficial for learners vulnerable to difficulty with fraction understanding (Shin & Bryant, 2015).

Additionally, the intervention research suggests that students gain from opportunities to learn from **multiple representations of concepts**. *Kickstart* lessons incorporate real world contexts and a range of examples. The *Kickstart: Fractions* books are set in the context of a bakery. Throughout the series, students work with ideas like portions of cake or brownies being sold or frosted, or batches of cookies and their ingredient measurements, and bread being baked and divided between friends. These authentic scenarios give students a chance to put meaning to abstract concepts like fractions.

Lastly, **cooperative discussion and the use of precise vocabulary** are important components of the *Kickstart: Fractions* approach that support conceptual knowledge. Explaining concepts verbally requires students to process their ideas more deeply and solidify their own understanding. Discussion about fractions helps students build understanding rather than memorize procedures without knowledge of why they make sense. By discussing how fractions relate to whole numbers, equivalent fractions, or operations like addition and subtraction, students solidify their own understanding.

Research indicates that students improve their math learning when they explain concepts and engage with others' ideas. Throughout each lesson, students regularly engage in peer and small-group discussion. Further, the program integrates techniques that support students' math vocabulary learning and retention. Use of precise vocabulary is important for solidifying math understanding and enabling students to effectively communicate their knowledge as they are learning (Riccomini et al., 2015). Throughout the program, students pair actions with new terms, a strategy for better long-term memory (Madan & Singhal, 2012).

Instructor language is scripted so that adults delivering the instruction use and promote the use of precise vocabulary (e.g., "numerator" rather than "top number"), which is essential for storage and retrieval (Powell et al., 2018). Students are consistently asked to utilize precise vocabulary, and instructors are encouraged to demonstrate that use of math language is valued through positive feedback statements and awarding Cory Coins. Intentional partnering and asking partners to use math language also provides students with opportunities to utilize their growing fraction vocabulary.

ADVANCE TO PROFICIENCY

Why Does Grade-Level Math Proficiency Matter?

Grade-level proficiency provides a benchmark to ensure that students acquire the essential mathematical skills deemed appropriate for their grade level. Proficiency at each grade level signifies that students have grasped foundational concepts, setting the stage for more advanced learning in subsequent years. Studies show that early math achievement learning gaps are difficult to close (Morgan et al., 2011), with impacts on academic and vocational outcomes in the long term (Geary, 2011). Demonstrating grade-level proficiency and having a strong foundation in mathematics at each stage of education contribute to a cumulative understanding of the subject, enabling students to tackle increasingly challenging math concepts.

“Mathematical concepts build on each other throughout the school years. A strong foundation allows students access to later mathematical concepts.”

— Jen Hunt, co-author of *Kickstart: Fractions*

Curriculum standards in grades 3–5 emphasize strengthening students’ fluency with whole numbers, fractions, and decimals, and applying these concepts to solve real-world problems. Fraction understanding is a critical foundation for future math achievement, students’ math related self-efficacy, and engagement in math learning in the intermediate grades and beyond (Siegler et al., 2012). Research exploring factors that predict successful outcomes with fraction learning show that the ability to place whole numbers on a number line—an indicator of fraction magnitude understanding—predicted success with fraction procedures at the end of fourth grade. Mastery of whole number estimation and the ability to visualize relationships between equivalent fraction quantities in fifth grade were among the key predictors of success with fractions in sixth grade. This work also shows that while early fraction understanding impacts math development during a critical period of instruction and influences overall math competence, a sizeable portion of children make limited progress toward grade-level proficiency with fraction concepts (Hansen et al., 2015; Jordan et al., 2013; Jordan et al., 2017).

Students who enter third grade without a firm grasp of number sense are more likely to approach fractions with memorization, rather than flexibility using conceptual reasoning (Geary, 2011; Kirkland et al., 2024), underscoring the importance of early

number sense and then continuing to develop this knowledge in the context of fraction learning (Kirkland et al., 2024).

Students who have encountered difficulty with fractions and are identified for general education intervention support are especially vulnerable for leaving elementary school without the grasp of fractions they need to make sufficient progress in advanced mathematics. However, students who receive timely and appropriately tailored math intervention are positioned to close critical gaps that interfere with their progress in core curricula. Fraction intervention studies also suggest that well-designed intervention can mitigate the negative effects of other factors, such as low attention (Barbieri et al., 2020).

How Does *Kickstart: Fractions* Help Students Advance to Proficiency with Fractions?

Kickstart: Fractions is a focused, systematic intervention program for students struggling with grade-level math skills due to challenges acquiring fraction concepts. By specifically addressing fraction understanding, which is foundational to overall math proficiency, the program provides a targeted approach to bridge gaps in understanding and quickly provide students with the foundation they need.

Kickstart: Fractions leverages an evidence-based graduated instructional sequence (concrete-representational-abstract) to build the strong conceptual understanding students need to meet grade-level expectations and progress toward proficiency. The program incorporates intentional practices that distribute learning over time and mix different types of problems or skills within practice sessions. These strategies help students retain information, strengthen understanding, and apply concepts flexibly in diverse contexts.

Placement checkpoints and progress monitoring tools allow instructors to identify students' starting points and gather data to adjust instruction as students advance through the lessons. Through this approach, students build a repertoire of skills and strategies that enable them to master concepts with greater ease and confidence.

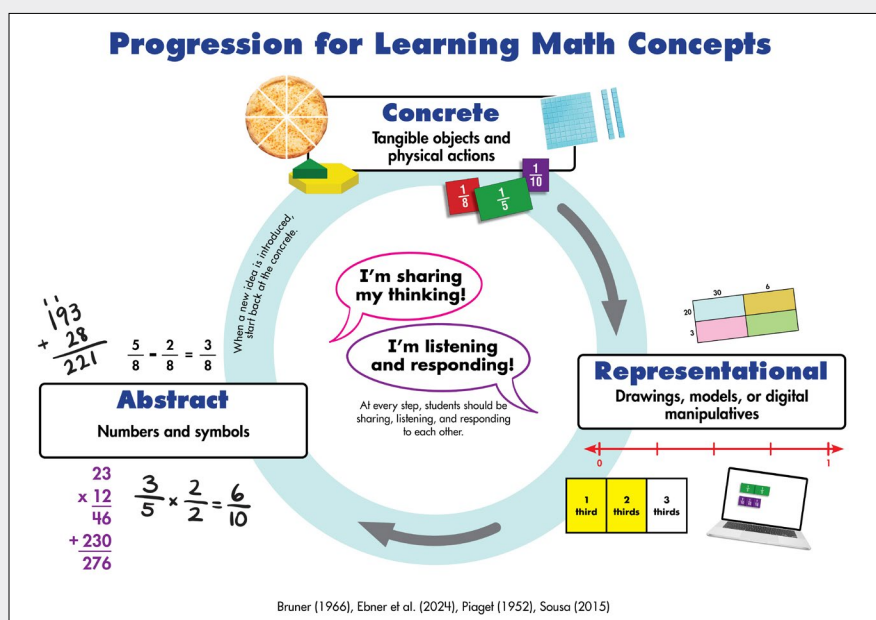
Deep Dive

Students at risk for math difficulties or who have experienced challenges with math learning and need support progressing to grade level require targeted, timely, evidence-based math intervention (Scammacca et al., 2020). Research shows that

using representations is an effective strategy for students facing math learning challenges. This work has demonstrated that students require clear instruction and guidance, particularly when they are expected to work with multiple types of representations (Jitendra et al., 2016).

In the context of teaching fractions, student support with representations can be offered through a clear and progressively structured sequence, utilizing the **concrete-representational-abstract (CRA) approach**. This evidence-based instructional framework systematically guides students in solving problems first with concrete materials, then using visual representations of those materials, and ultimately solving problems abstractly, without any physical or visual aids (Mercer & Miller, 1992). The CRA approach has roots in cognitive development (Bruner, 1966; Piaget, 1952) and constructivist learning (e.g., see Schunk, 2012) theories, and leverages work informing instructional scaffolding (e.g., Zone of Proximal Development, Vygotsky, 1978). This approach meets students at their current level and helps them link new information with what they already know, leading to better retention.

A recent meta-analytic review of the CRA math approach implemented with diverse populations, settings, and target skills (with most studies conducted in grades K–5) showed a large, positive effect on student math achievement and a benefit to children within both special education and general education classrooms (Ebner et al., 2024). Studies also demonstrate specific benefits of this approach for teaching fractions (Flores et al., 2018; Shin & Bryant, 2015), especially when combined with explicit instruction (Hudson et al., 2006) and application of concepts within word problems with culturally relevant teaching examples (Kim et al., 2015).



Fractions are inherently abstract, and a CRA approach allows students to first understand fractions in a concrete, relatable way before moving to symbolic math. As such, the CRA progression is a central part of the *Kickstart: Fractions* approach. Multiple representations help students build both conceptual and procedural knowledge. By design, the program presents concepts using a progression that moves students from concrete understanding to representational to abstract with explicit instruction. This progression is presented daily and throughout concepts to help students build their deep conceptual understanding of fractions.

At the **concrete phase**, students use tangible objects and physical actions. At the **representational phase**, they use drawings, models, or digital manipulatives. Then they move to representations with numbers and symbols at the **abstract phase**. At every step, students are sharing, listening, and responding to each other. This cycle is repeated for each new idea. As each new concept is introduced, the learning progression begins again at the concrete and moves students to the abstract systematically.

This math learning progression highlights how all learners begin with rudimentary understanding of concepts through the concrete. This concrete practice helps students encode conceptual understanding in the brain. Once encoded, it can later be used representationally, and following additional practice, abstractly. For example, as students are introduced to fractions in Book 1, Lesson 2, they are given standard pattern blocks to reason about relationship between parts (triangles) to a whole (hexagon). That same day they also utilize unlabeled fraction bars to make the same relationship observations using a different manipulative. By Lesson 4, students are working with fractional concepts in images of length and area models. Some lessons have students standing in groups to physically embody fractional portions.

Throughout, the instructor is pairing these concrete and representational models with the written fractions to give students a framework for the abstract. That way, in Lesson 6, students begin writing fractions with understanding. As students progress to Book 2 (addition and subtraction of fractions), the CRA cycle begins again; students pair these actions with drawings and fraction notation. In Book 3, students practice the multiplication and division of fractions, and once again, students are provided fraction bars and pattern blocks and draw before they use notation so that deep conceptual understanding leads the procedural moves.

This developmental progression acknowledges that learning is not one-size-fits-all and provides a scaffolded approach to effectively support students in mastering challenging concepts. It allows instructors to meet students where they are and help them connect new and known information for stronger retention. When math concepts are taught systematically, students are able to build learning incrementally; they can leverage their prior knowledge to learn new content (Fuchs et al., 2021a).

With a structured process that helps build a deeper understanding of fractions, students begin to see the “why” behind the procedures. They recognize patterns that they might have missed if the instruction had skipped the concrete and representational phases and gone directly to the abstract. This approach facilitates the fluent application of skills and reduces confusion as students progress to more complex math concepts later.

Kickstart: Fractions also incorporates intentionally designed practice to strengthen learning and retention by ensuring that topics are revisited systematically over time. This promotes transfer and long-term mastery of concepts learned. Mass, or blocked, practice is characterized by introducing and practicing one concept for a long period of time. However, research has shown that mastery and long-term retention are significantly improved when practice is spread out and mixed with other tasks, rather than when the practice involves repeating the same task or material multiple times in a row (Brown et al., 2014).

Throughout the program, *Kickstart: Fractions* presents concepts and skills that are repeated and gradually increase in difficulty, presenting new learning opportunities that are connected to previous learning and prior knowledge. For example, the progression in Book 2 has students learning to add and subtract fractions beginning with the basic concept of adding unit fractions. Lessons 1–4 give students the time and tools they need to deeply understand how to add and subtract unit fractions. This provides a foundation for Lesson 5, where they add non-unit fractions. Lessons 12 through 16 teach students to apply their knowledge to add and subtract mixed numbers without regrouping, and this practice allows them to use those skills to add and subtract mixed numbers with regrouping starting in Lesson 17.

Additionally, consistent with the approach of all *Kickstart* products, *Kickstart: Fractions* integrates intentional cycles of review and varied practice into its instructional design. The nature of this approach differs slightly from other *Kickstart* products, with *Kickstart: Fractions* using smaller doses of variation to ensure

sufficient time for practicing the skills essential for fraction understanding, while systematically progressing from concrete to abstract.

Throughout the program, instruction frequently circles back from abstract concepts to concrete representations before moving forward to the next topic, which again transitions from representational to abstract phases. This cyclical process aligns with research-supported methods that promote deep and lasting conceptual understanding (Kang, 2016; Rohrer et al., 2020). If students struggle with abstract tasks at the end of a stage, instructors are encouraged to revisit earlier lessons that were modeled in the concrete and representational phases to reinforce understanding and build confidence.

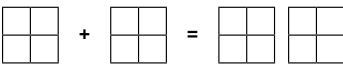
In addition to a systematic developmental progression embedded in the instruction, **partner driven, high-quality math tasks** delivered in short bursts

are integral to the instructional design of *Kickstart: Fractions* that advances students to grade-level proficiency. While there is limited research prescribing the specific duration of supplemental math interventions, new findings point to short, more concentrated interventions as a potentially more time-efficient way to intervene for students who are at risk for math failure (e.g., DeFouw et al., 2023).

The 20-minute lesson design of *Kickstart: Fractions* promotes consistent, short bursts of instruction that holds students' attention and promotes engagement necessary for effective learning. Instruction also leverages several characteristics of high-quality math tasks, including authentic, interesting examples that encourage students to make connections between concepts, apply different strategies for finding solutions, and foster opportunities for students to talk about their math

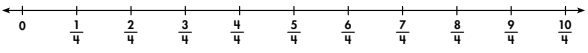
Kickstart: Fractions Book 2
Lesson 6 Practice Name: _____

At the end of the day at the bakery, Gloria collects all the leftover treats and takes them to a home for elderly birds. Today at closing, $\frac{3}{4}$ of the strawberry cake was left, and $\frac{3}{4}$ of the banana cake was left. How much cake was left? Show your answer two ways, as a fraction greater than one and as a mixed number.

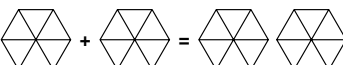


Fraction greater than one:

Mixed number:

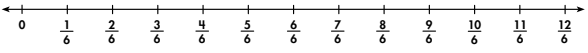


On the way to work at the candy company, Cory left home and walked $\frac{5}{6}$ of a kilometer to the coffee shop, then $\frac{4}{6}$ of a kilometer to get from the coffee shop to the candy company. How far did Cory walk? Show your answer two ways, as a fraction greater than one and as a mixed number.



Fraction greater than one:

Mixed number:



Kickstart: Fractions Book 2
Reproducible 14
Lesson 6 Practice

learning (SanGiovanni, 2016). As shown in the excerpt below, discussion promotes students' ownership of math and investment in their learning.

T: What is your way of finding the whole in a fraction greater than one? There are a few ways to do it. Do you visualize it as pattern blocks? As fractions bars? As a number line? Do you think about how many wholes are in the numerator? Talk with your partner for a moment about what works for you.

This discussion is essential. Students claiming a math method as their own way gives them ownership of math and investment in their learning. It's also okay if students aren't sure yet, but encourage those students to think about which method they prefer. This will nudge them toward ownership. Collect pattern blocks. Hand out pencils and Lesson 6 Practice. (R14, see Figure 6.8)


Further, the integration of student discussion gives learners the opportunity to process the concepts through language, as areas of the brain involved in language processing are essential to math learning (Peng et al., 2020; Sousa, 2015). Spoken language is particularly important for math learning. Studies show that a student's competence with academic discourse is a critical prerequisite for constructing math knowledge (Erath et al., 2018) and that when students improve their language skills for collaborative reasoning, it enhances their individual learning and understanding of math (Mercer & Sams, 2006).

Throughout *Kickstart: Fractions*, students are taught to use **precise mathematical language** and engage in regular **peer and small-group discussions**, which fosters their use of language to effectively communicate their understanding. In each lesson, partners are asked to share their approaches, strategies, or estimations. Typically, students are given a moment to think of their own answer or idea, then one of the two partners is asked to share: "Lemons, tell your Limes a fraction greater than one-fourth. Limes, if they are correct, you will name another fraction greater than one-fourth." This requires each partner to actively engage in the learning and keeps students from opting out.

Importantly, *Kickstart: Fractions* provides instructors with Placement Checkpoints to ensure appropriate placement within the program and to support the ability to efficiently collect data that helps them fine tune their instruction. Intentional placement and grouping in the program are designed to promote efficiency of instruction. Accurate placement ensures students will be learning what they need for advancing to grade-level proficiency and not wasting time reviewing material they may already know.

Placement Checkpoints can be readministered to provide pre- and post-test data for each stage, giving instructors insight into any remaining gaps in knowledge that need to be addressed. Additionally, instructors collect valuable information about students' progress during the **Landing phase** of each lesson. These can serve as "exit tickets" to monitor what students have learned in response to the instruction and can be tracked on the included daily progress monitoring sheet, which offers a clear view of each student's growth throughout the program.

Students are also given regular opportunities to reflect on their math understanding, offering instructors important formative assessment data and insight into the student's self-perception as a mathematician. This information can help guide focused interventions. Recent research highlights the value of student self-reflection in formative assessment, emphasizing the importance of involving students actively in their own learning to enhance the effectiveness of formative assessment in interventions (Lee et al., 2020).

Today's learning target:	Self-Reflection:
 I can shade, order, and name mixed numbers and fractions greater than one.	<input type="radio"/> I can teach this to another student. <input type="radio"/> I can do this on my own. <input type="radio"/> I can do this with some more practice.

STRENGTHEN POSITIVE MATH IDENTITY

Why Does a Student's Identity as a Mathematician Matter?

A positive math identity refers to how an individual perceives themselves and their belief in their capability to engage with and succeed in mathematics. Believing in one's own capability to understand and perform math is closely linked with growth mindset, where students believe their academic abilities can improve with effort and practice. Research has consistently demonstrated a connection between students' self-perceptions about their math ability and their later math performance (e.g., Bohrstedt et al., 2021; Fernandez et al. 2022; Petersen & Hyde, 2015).

Students who view themselves as capable in math often show greater persistence, achievement, and engagement in school, indicating that developing a positive math identity can be a critical factor in enhancing math performance. In addition to the association with academic outcomes in math, students' math identity has been shown to be a strong predictor for career choice in STEM fields (e.g., Cribbs et al., 2021).

Students' underlying feelings and emotions (their affective state) impact their approach to math problems as well as their willingness to engage with the material and maintain effort when challenges arise. Harder & Karber (2024) summarize research describing two contrasting emotional pathways that can impact a student's learning and mathematical identity. As described, students may experience a positive affective cycle, in which they approach challenges with curiosity and motivation, exploring different strategies despite moments of frustration, ultimately leading to a deeper understanding and feelings of satisfaction, relief, and a boost in confidence. This in turn strengthens their math identity. In contrast, students in a negative cycle may begin with curiosity but limit their exploration to familiar methods and if they fail in their attempt, experience frustration, anxiety, and hopelessness that causes them to give up.

Successful math students tend to demonstrate a positive attitude toward **productive struggle** and the resources to persevere through complex tasks. In fact, research exploring mathematical resilience—an optimistic mindset toward mathematics that helps learners overcome emotional challenges encountered during the learning process—suggests that learning environments that foster math resilience enable learners to have agency, independence, and choice, allowing them to engage with tasks that challenge them (Lee & Johnston-Wilder, 2017).

Research examining math identity and student motivation suggests that educators can influence students' math identities. Intervention studies find that teachers who actively support students in exploring their self-perceptions about math can help foster positive math identity and enhance motivation for learning math. Further, when teachers create a classroom environment where students are encouraged to explore how math concepts relate to their belief in their ability to do math, students are more likely to develop positive and adaptive math identities (see Heffernan et al., 2020 for review).

Notably, research exploring math anxiety finds that teacher math anxiety relates to lower achievement among students and that student perception that their teacher holds a fixed mindset about math helps in part explain this relationship. Additionally, this work demonstrates that teachers' use of process-oriented (as opposed to ability-oriented) teaching practices were more powerful than their usable knowledge for teaching math in shaping students' math perceptions. In short, the way teachers feel about math and the subtle cues they communicate through their teaching may significantly influence student math learning (Ramirez et al., 2018).

How Does *Kickstart: Fractions* Strengthen Math Identity?

Kickstart: Fractions intervention lessons and instructional materials are intentionally designed to strengthen students' identity as mathematicians by fostering a positive learning environment, encouraging student ownership of their learning, and empowering instructors to feel capable in delivering effective math instruction. Instruction emphasizes building deep conceptual understanding to position students to be more successful in their math learning, fostering a sense of confidence and competence. Lessons are also designed to equip instructors to share positive views and growth-oriented mindsets about fraction learning, while also ensuring students experience moments of joy, feel respected sharing their math thoughts, and feel valued as math learners.

Deep Dive

Creating an **encouraging, supportive math learning environment** that builds students' understanding of and ownership over the math they are learning is an integral part of the *Kickstart: Fractions* intervention design. As described in sections above, the program centers on helping students (and instructors) understand the "why" behind the math. This is accomplished through lesson content and activities that focus on building the conceptual understanding that underlies fraction concepts and procedures. This is important given the impact of self-concept on math learning. For example, students with strong math performance tend to have more positive math self-perception, greater enjoyment in math, and less math anxiety than their lower achieving peers (e.g., Van der Beek et al., 2017).

As students increase their conceptual knowledge, they are more likely to be successful with fraction problems both in the classroom and in real-life situations, which helps them feel more confident and capable as mathematicians. Program materials also guide instructors to praise positive, adaptive learning behaviors they want to see and to encourage students to persevere when challenged. Further, Warm-Up activities and the brief, consistent opportunities for students to share their ideas with a partner that are built into the lessons help foster a supportive classroom culture.

The program was also intentionally designed to promote **positive perceptions about math and reduce math anxiety**. It regularly assists in student's development as mathematicians by normalizing and encouraging practices that mathematicians

use, such as: 1) persevering when confused, 2) using peer ideas as a springboard, 3) revising their thinking, and 4) viewing mistakes as opportunities to learn. In this way, instruction is designed to build growth-oriented mindsets about mathematics in general, and fraction learning more specifically. Lessons and materials foster collaboration among learners, and make the content approachable and manageable through meaningful, relevant examples, manipulatives, and built-in time to warm up each day.

The structure and approach written into the format of the *Kickstart* lessons addresses several areas that can contribute to math learning difficulties. Specifically, research suggests that a student's perceived lack of control over their ability to succeed in math is associated with a negative attitude toward the subject (Zan & Di Martino, 2009) and that self-perception of math competence is more strongly related to math anxiety than actual math achievement (Li et al., 2021). Throughout *Kickstart*, there are repeated opportunities for students to solve the Landings (activities at the ends of lessons to revisit the learning target), work with partners, and show their work in various modalities (e.g., manipulatives, drawings, equations).

Students are also given small, manageable tasks to do without the instructor stepping in. These occasions provide students with numerous opportunities to take charge of their learning and reduce feelings of learned helplessness. Students who experience repeated failure or a lack of control over their learning success can begin to become dependent on the teacher, contributing to a passive learning cycle where students show diminished active participation or self-regulation of their own learning (Miller & Mercer, 1997).

To prevent passivity, *Kickstart: Fractions* utilizes ample partner work; students are assigned a partner and equally share the responsibility of talking. Additionally, through accessible activities, experiences of success, and a sense of belonging within the math learning context, students are more likely to feel equipped to engage successfully and less inclined to avoid participation.

Memory difficulties are another consideration embedded into the *Kickstart* instructional design. Attention and memory play a critical role in supporting math learning, and difficulties with working memory contribute to challenges acquiring procedural skills and can impede retrieval of information (Geary, 2011). To support students' storage and retrieval of the math concepts introduced, *Kickstart: Fractions* makes use of **techniques known to support memory**, such as spacing, gesture,

manipulatives, and revoicing (Beilstein, 2019; Carbonneau et al., 2013; Carpenter et al., 2022; Ferris, 2014).

The program also fosters students' focus on the instruction through hands-on work and pacing designed to keep them moving along and engaged, while reducing boring or mundane tasks. Likewise, the Learning Target, a part of the daily instructional routine, is designed to ensure that instructional goals are clearly communicated to students and that their attention is drawn to learning goals and what they are expected to learn, which can help them stay focused and feel more positive about engaging with the material (Serki & Bolkan, 2023).

Kickstart: Fractions was designed with the understanding that math anxiety is a real and significant challenge, particularly for students who need targeted and explicit instruction to build both math skills and a positive mindset toward math. Each aspect of the program was crafted to support student success and alleviate math anxiety. A wealth of research has demonstrated the importance of **teacher attitudes** as a critical factor in shaping student attitudes toward mathematics (e.g., Beilock et al., 2010; Blazar & Kraft, 2017; Russo et al., 2020).

Kickstart lesson scripts encourage instructors to approach students with the mindset that students want to succeed in math, even when they face challenges and to consider how a teacher's positive attitude about math and a focus on what students can do, influences their own self-perceptions. In fact, studies exploring teacher and student characteristics influencing math learning such as the one by You et al. (2021) show, among other factors, the importance of the student-teacher relationship. *Kickstart* also addresses the important role of the curriculum in influencing math anxiety. There is a deliberate focus within the instructional approach to engage and connect various brain systems to enhance students' retention of information based on what is known about the role of different areas of the brain involved in math learning (Hinton et al., 2012; Martinez-Lincoln et al., 2023; Sousa, 2015). Additionally, the use of hands-on, partner-driven work in short 20-minute bursts is purposefully incorporated to support student's needs (e.g., Codding et al., 2016; Klang et al., 2021; Theobald et al., 2020).

Specific to the **instructional strategies**, *Kickstart: Fractions* integrates a motivational structure, lesson set-up, and placement approach that carefully considers the age and developmental stage of the students participating in math intervention and intentionally builds these strategies into learning activities to offer consistency

and experiences of success through **collaborative team activities such as the CRANE-ium Competition**. This is designed to promote a team mentality and to give students that chance to be **rewarded for positive participation** instead of accuracy. Likewise, throughout, **mistakes are celebrated and reframed as valuable learning opportunities** to disrupt beliefs that math must be done correctly on the first try. *Kickstart's* embedded motivational structure aligns to research showing that interventions based on motivational theory that emphasize motivational processes as key to improving student learning outcomes have demonstrated effectiveness (Lazowski & Hulleman 2016).

Furthermore, the safe, supportive **classroom culture** created through features such as the Warm-Ups and daily partner talk serves to help alleviate math anxiety given what is known about the way stress can interfere with or alter learning (Vogel & Schwabe, 2016) and the way in which math anxiety negatively impacts math achievement and students' feelings of competence (Ramirez et al., 2013). Math anxiety can create a cumulative effect that interferes with math learning by altering students' attitudes and motivation, contributing to math avoidance, and disrupting cognitive processing during problem-solving (Ramirez et al., 2013). Research also underscores the importance of educational interventions that focus on managing negative emotional reactions to math-related stimuli (Lyons & Beilock, 2012).

Program Note

Students at this level of intervention have often spent years confused in math class. They have commonly been given the impression that being wrong in math is bad. Too few students have had an adult say, "Mistakes are opportunities!" After all, the moment a mistake is realized can be a powerful opportunity to learn new information. The brain is primed to receive and store new input. That is a great thing! Perhaps students would benefit from hearing, "You tried. Thank you for that effort. This time there was an error in your thinking, which means you are ready to learn!"

Unlike formal assessments, the program's placement checkpoints provide an alternative approach to reducing math anxiety and fostering positive math identities. Instructors are encouraged to use these checkpoints in a pre- and post-test structure, one-on-one, incorporating manipulatives and engaging learners in open, conversational dialogue about their thinking. This approach allows students to demonstrate their knowledge more freely, minimizing math anxiety and fear of outcomes (Lyons & Beilock, 2012; Pérez-Fuentes et al., 2020).

Lastly, *Kickstart: Fractions* was written with all types of **instructors** in mind. Lessons are scripted so that an adult, regardless of math teaching background, can successfully implement the instruction. The format of the teacher text outlines specific ways instructors can relate mathematical ideas, make connections between concepts, and support learners in developing their math identity. The explicit nature of the text does not require that an instructor is comfortable with math concepts in order to provide high quality, structured intervention. In this way, it is possible for the adults to increase their confidence and discover their “math voice” alongside students.

Are You Ready to Improve Your Students' Fraction Understanding?

Use these questions to reflect on your current practice, considering the research featured in this paper.

- How do you currently provide math intervention to students who have difficulty with fractions?
- Are your students engaged in interactive and motivating fraction activities?
- Do lessons provide ways to ensure students develop deep conceptual understanding and procedural skills?
- Do students have opportunities to leverage a systematic, developmental progression and apply concepts to everyday examples to build strong understanding and proficiency with fractions?
- Do students understand the “why” behind fraction procedures?
- Are intervention activities explicit, meaningful to students, and effective in helping bridge new and known information?
- Does intervention foster collaboration and a positive, supportive math learning culture where students are able to take ownership of their learning, value opportunities for growth, and see themselves as mathematicians?
- Does instruction foster positive perceptions about math learning and math self-concept among students and instructors?

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