LXD RESEARCH
ESSA Level 4 Evidence

LEARNING EXPERIENCE DESIGN | LXD RESEARCH | ZANER-BLOSER

Zaner-Bloser *Building Fact Fluency* **Foundational Research Paper**

RESEARCH-BASED DESIGN & LOGIC MODEL

Zaner-Bloser *Building Fact Fluency* deepens students' conceptual understanding of arithmetic operations, increasing procedural fluency, bolstering access to rich problem-solving experiences, and strengthening identity as mathematicians to build community, curiosity, and confidence in math learning.

Zaner-Bloser

Learning Experience Design (LXD) Research & Consulting
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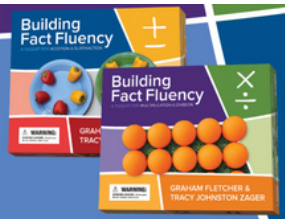
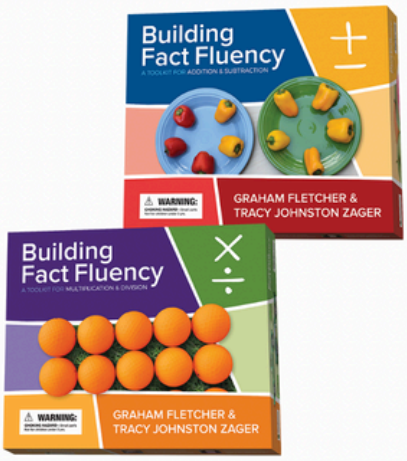
Building Fact Fluency



Build Fact Fluency Through Real-World Contexts and Purposeful Practice



Building Fact Fluency toolkits (*Addition & Subtraction* and *Multiplication & Division*) help students develop deep conceptual understanding of the operations and fact fluency at the same time.

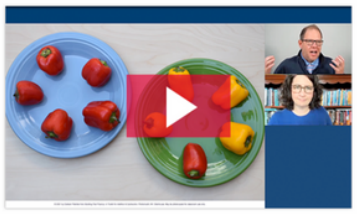
Research-based and standards-aligned, each toolkit invites students to think strategically about mathematics through multiple, rich, real-world contexts.

These accessible contexts allow students to

- see how number facts connect to a wide variety of mathematical situations
- explore the properties of the operations
- build a foundation of strategies they can draw from efficiently and with confidence.

Who *Building Fact Fluency* Is For

Each *Building Fact Fluency* toolkit is an ideal supplement to any curriculum or for any situation where students need additional fact fluency practice and review.



Program authors introduce *Building Fact Fluency*.
Watch Video

- ✓ K–5+ Classrooms
- ✓ Intervention
- ✓ Multilingual Learners
- ✓ Special Education
- ✓ After School, Summer School, and Tutoring

Building Fact Fluency teachers are members of a robust Facebook Community of educators who are passionate about math education. Members support each other with tips on implementation and getting the most out of the kit and online resources. Anyone looking to connect with peers on how to deepen students' mathematical understanding through meaningful instruction is welcome to join!

ZB

Building Fact Fluency Authors Page

The Building Fact Fluency author team combines years of classroom experience with a passion for helping students develop deep, conceptual mathematical understanding through purposeful practice.

Tracy Johnston Zager

Tracy has worked in many schools throughout her career, first as a fourth-grade teacher, then as a supervisor of preservice teachers and their in-service mentors, and currently as a math coach. In addition to Building Fact Fluency, Tracy is the author of Becoming the Math Teacher You Wish You'd Had: Ideas and Strategies from Vibrant Classrooms.

Graham Fletcher

Graham Fletcher has served in education as a classroom teacher, math instructional lead, and currently as a math specialist. Graham's work with math progressions and problem-based lessons has led him to present throughout North America and beyond.

UNDERSTANDING
ESSA Evidence

Evidence guidance under the Every Student Succeeds Act (ESSA) are designed to ensure that states, districts, and schools can identify programs, practices, products, and policies that work across various populations.

The Every Student Succeed Act (ESSA) requests that education programs provide evidence of effectiveness and impact in order to be federally supported. The Department of Education's Office of Educational Technology provides standards to assess the varying levels of strength of research for education products.

The categories for ESSA Evidence are: strong, moderate, and promising evidence of effectiveness or demonstrates a rationale to be effective.

This product meets the requirements for Level 4: Demonstrates a Rationale

- ✓ Includes a logic model based on research
- ✓ Research documentation connects academic research studies to features in the product that support learning
- ✓ A study is planned and/or currently underway
- ★ A third-party research organization has reviewed the documentation for ESSA validation

When product designers leverage learning sciences to design their products, educators can better target instruction, and students' skills soar. Through interviews with the product designers and an evaluation of their research-informed activities, this product meets the criteria for LXD Research's ESSA Level 4 Evidence.

– Rachel Schechter, Ph.D., Founder of LXD Research

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Zaner-Bloser Building Fact Fluency

Foundational Research Guide:

Research-based Design of Building Fact Fluency

Zaner-Bloser Building Fact Fluency deepens students' conceptual understanding of arithmetic operations, increasing procedural fluency, bolstering access to rich problem solving experiences, and strengthening identity as mathematicians to build community, curiosity, and confidence in math learning.

Outline

How Can This Research Foundations Paper Be Used?

This foundational research paper aims to explain the innovative strategies, pedagogical approaches, and empirical evidence that inform this program's design. This paper answers, "How was research used to create this program?" by narratively describing the program's theory of action. The program components and its intended outcomes are framed in a logic model required as part of the Every Student Succeeds Act (ESSA). While the details of what is required for each federal funding program are slightly different (Non-Regulatory Guidance document), the information in this document should satisfy the requirements for "demonstrates a rationale based on high-quality research findings or positive evaluation that such activity, strategy, or intervention is likely to improve student outcomes or other relevant outcomes; and includes ongoing efforts to examine the effects of such activity, strategy, or intervention." Each section in this paper details how *Building Fact Fluency* engages students in learning, deepens their understanding of math concepts and strategies, transfers and extends these concepts and strategies to novel situations, and strengthens their ownership of math learning.

Key Points

- Procedural fluency in mathematics is fundamental to learning math with deep understanding. To be computationally fluent, students not only need to be accurate in finding answers but also efficient and flexible in their problem solving and application of strategies.
- Fluent retrieval of basic arithmetic facts is the cornerstone of procedural fluency. Students need to be able to recall single-digit facts accurately, automatically, and flexibly to support their ability to solve complex problems.
- Building Fact Fluency enables students to apply strategies that connect number facts to a wide variety of mathematical situations, avoiding a singular focus on rote memorization, which has been shown ineffective.
- Building Fact Fluency promotes student engagement through strategy discussions, contextualized practice, reflection, and engaging games.
- Building Fact Fluency's scope and sequence are aligned with Bay-Williams & Kling's (2019) flexible learning progression, which engages students to explore and work with foundational facts before derived facts are layered in.
- Building Fact Fluency's toolkit leverages multiple contextual themes presented per strategy, which increases students' ownership of their math learning
- By presenting a range of strategies, students will have more autonomy in choosing strategies and facility in transferring and extending their fact knowledge. .

What is Fact Fluency, and Why Does it Matter?

Math fact fluency, the ability to quickly and accurately recall basic addition, subtraction, multiplication and division facts, is the foundation of math success and lays the groundwork for the ability to solve increasingly difficult, multi-step math problems. Math fact fluency is important especially among young learners because it establishes a foundation for success in higher level math instruction, builds confidence in math ability, and enables students to solve increasingly difficult and complex problems more quickly and with greater accuracy. Students who are fluent in their retrieval of basic facts are able to choose flexibly among methods and strategies to solve contextual and mathematical problems—they can understand and explain their approaches and produce accurate answers efficiently (National Council of Teachers of Mathematics, 2014).

Fact fluency is not just a prerequisite to more advanced problem solving; it is also an outcome of rich problem solving and purposeful practice. When students have sufficient, meaningful opportunities to make sense of addition, subtraction, multiplication and division, they start figuring out what's true about these operations. When students have a strong understanding of basic math facts, they can use what they already know to figure out new facts and understand more complex math concepts, as compared to the idea that students must first memorize all their basic facts before they can start learning more advanced math (Van De Walle et al., 2019).

Strong fact fluency means that students can ultimately retrieve facts efficiently across the wide variety of mathematical situations, problems, and contexts. In addition to the available evidence showing the importance of fact fluency, the authors of Building Fact Fluency have also come to understand three things clearly through experience:

1. Students need to be able to know and use their facts in the midst of more complicated problems without losing track of their thinking.
2. Many students who can say their single-digit facts do not use them effectively or flexibly when solving problems.
3. Students who are not given enough opportunities to uncover the properties of numbers and operations have trouble identifying relationships and making meaningful connections later on.

Logic Model for Building Fact Fluency

PROBLEM STATEMENT

Many elementary school students struggle with achieving math fact fluency, and teachers experience challenges with increasing math fact fluency in their students. Although online tools can support memorization of basic math facts, they often lack opportunities for students to exercise math flexibility and participate in peer discourse on mathematics.

RESOURCES

What resources are or could be available?

- Dedicated time for small group or paired activities during the math period
- Expert-developed learning content
- 3-act tasks (problem-based learning)
- Manipulatives
- Game boards
- Teacher's guide (lessons, language, directions)
- Additional support materials
- Math instructional coach to support content integration and teacher training
- A school model for assessment and providing intervention support to students as needed
- Access to a device, such as a laptop, and the internet for online parts of the program

STRATEGIES & ACTIVITIES

What will the activities, events, and such be?

- A structured approach to developing math fact fluency
- Lesson strings, including core and optional routines, to optimize relational thinking.
- Problem-based learning that enables students to apply their mathematical abilities to real-world situations and interactions
- Game-based learning that increases student engagement and supports an understanding of complex concepts
- Practice questions and immediate feedback to aid in student learning
- Students self-assess and verify the accuracy of their own work
- Teachers use formative assessments to understand student skill progression

OUTPUTS

What are the initial products of these activities?

- Teachers deliver more interactive lessons.
- Teachers reduce students' time in rote memorization or fact drills; instead build math fluency in meaningful contexts.
- Teachers hear more student discourse and learning of math fact fluency in the classroom.
- Students engage in activities that foster relational math thinking.
- Students engage in hands-on scenarios, which helps them to be more prepared for real-world situations. Students engage with activities that align with their skill level, addressing their unique learning needs.
- Students gain opportunities to self-assess, verify their accuracy, and revise their own work.
- Students perform retrieval practice through inquiry-based learning with peers.

SHORT-TERM AND INTERMEDIATE OUTCOMES

- Teachers build capacity in math instruction to support individual student growth.
- Teachers more successfully differentiate instruction for all learners.
- Students improve their math fact fluency.
- Students build language relationships and math understanding through discourse.
- Students find their math learning more relevant.
- Students retain and apply math learning in novel situations.
- Students are less likely to hesitate to complete math tasks.
- Students are more likely to be confident in their math abilities.

LONG-TERM OUTCOMES AND IMPACTS

- Students develop relational thinking that applies to many aspects of their lives and can be built upon as they progress through higher-level mathematics.
- Students are more creative and tenacious in solving problems.
- Students are more likely to consider a future career in math-based fields.
- Students are more likely to feel comfortable in the math community.

ASSUMPTIONS

- All teachers attend professional learning events.
- The program is implemented fully and consistently.
- Product activities are aligned with the overall math progression at the school and are not an isolated experience.

ZB Building Fact Fluency

The Goal of Zaner-Bloser Building Fact Fluency

Instructional Approach

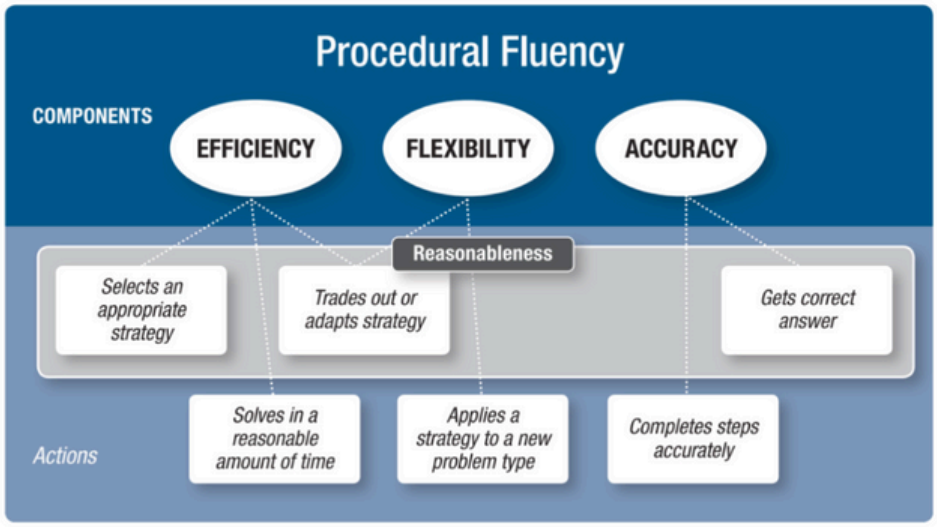
Building Fact Fluency is an instructional toolkit designed to help teachers build students' conceptual understanding of math operations through rich tasks, strategy discussions, contextualized practice, reflection and engaging games. It was designed as a resource that puts existing research on fact fluency to use in a practical and accessible way. Building Fact Fluency is structured and sequenced so that students come to know and apply their facts by noticing relationships and making connections. Rather than focusing on rote memorization of discrete facts in a vacuum, the goal of Building Fact Fluency is to provide students with the time and ample, varied opportunities they need to both acquire and flexibly apply number fact strategies while discovering fundamental properties of arithmetic in order to learn math with deep understanding.

Building Fact Fluency employs Lesson Strings—a series of interconnected activities, tasks, and games—to provide students with ample opportunities to apply mathematical operations in diverse real-world contexts. These Lesson Strings, with their large variety of different contexts, are designed to advance procedural fluency among students. Procedural fluency refers to the knowledge of procedures, the knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently (National Research Council, 2001).

> "A big part of shifting the way we teach fact fluency is shifting the way we think about the relationship between fact fluency and conceptual understanding. They don't need to be separated and one doesn't need to wait for the other. Building Fact Fluency is structured and sequenced so that your students will accumulate their known number facts at the same time they are developing understanding of the operations and discovering the fundamental properties of arithmetic."
>
> -Graham Fletcher & Tracy Johnston Zager

Within Lesson Strings, students engage in contextualizing and decontextualizing numbers, facilitating the construction of connections across mathematical ideas intended to support procedural fluency (Fosnot & Dolk, 2001). When students engage in the fluent retrieval of basic facts, they build a foundation of procedural fluency. This foundation of procedural fluency allows them to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures, and to recognize when one strategy or procedure is more appropriate to apply than another (National Council of Teachers of Mathematics, 2014). These contexts, serving as the central organizing structure of Zaner-Bloser Building Fact Fluency, transcend mere word problems or backdrops, presenting authentic problematic situations that resonate with students, stimulate the exploration of mathematical concepts, and support the process of engaging in mathematical thinking.

Image 1. Procedural fluency components and actions.



Source: Adapted with permission from D. Spangler & J. Wanko (Eds.), *Enhancing Classroom Practice with Research behind Principles to Actions*, copyright 2017, by the National Council of Teachers of Mathematics. All rights reserved.

Program Components

Building Fact Fluency provides students a deepened understanding of operational relationships by talking and describing their thinking and the varied ways in which to apply that knowledge.

Specific program components include:

- Math Dialogue and Discussion
- Game-based Learning/Collaborative Tasks
- Assessment Opportunities

These rich, truly challenging learning opportunities allow students to generate and explore mathematical ideas at multiple entry points, which supports mathematizing (Fosnot & Dolk, 2001). As they progress, students are able to leverage multiple strategies to answer questions, an indicator of being truly fact fluent.

Math Dialogue and Discussion

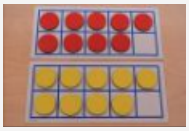
Zaner-Bloser Building Fact Fluency includes multiple opportunities, through Lesson Strings, for students to engage in math dialogue and discussion. By conversing with others about vibrant images using both informal and mathematical language, students are building not only their fact fluency, but their skills in reasoning about mathematical situations, selecting appropriate strategies and defending their choices, and drawing on prior knowledge and related vocabulary. These opportunities are particularly helpful when working with students who are English learners or are emerging readers, because vocabulary can be introduced informally through these conversations about images. When presented with unfamiliar contexts, conversations may help students begin to make sense of the scenario. In order to move conversation along and invite as many students as possible into math conversations, educators also have the opportunity to ask students about their noticings and wonderings. Once students realize there are no right or wrong answers to these questions, educators should expect to see an increase in participation. Providing opportunities for students to notice and wonder about an engaging image gives a chance to hear more students' brilliant ideas.

Image 2. Example of a Lesson String

#1 Crayons
Sums within 5

Image Talks

Anchor Problems



Tool Talks

Contextualized Practice Problems

Number Talks
(Optional in kindergarden)

Games

3-Act Math Task

English | Spanish

Image 3. Example of an Image Talk video

Game-based Learning & Collaborative Tasks

Building Fact Fluency contains a number of games and collaborative tasks designed to give students meaningful opportunities to practice their number facts and think strategically. The games and collaborative tasks were designed for both student autonomy and mathematical decision-making. When playing these games and interacting with peers, students are deeply engaged, which presents an optimal opportunity for educators to listen in, observe, and confer. Furthermore, the games have been modified to provide as much differentiation as possible. For example, there are games for each foundational and derived strategy in the Addition and Subtraction Toolkit, as well as included game board variations for combinations within 5, 10, and 20.

"Playing games are a primordial aspect of what it means to be a child and they develop within a motivating environment; therefore, not to take advantage of games as a learning resource would be to neglect an important asset."

-Campos, H., & Moreira, R. (2015).

Assessment Opportunities

As teachers implement the included assessment opportunities in Building Fact Fluency, they can systematically listen to students and collect information on the development of their thinking. The Building Fact Fluency Toolkit offers multiple opportunities for teachers to assess their students' understanding, thinking, and emerging fluency.

Teachers can pull up a chair and sit with students while they work on a problem; students can talk with their peers about their thinking; or groups of students can play a variety of games to practice their skills. The variety of opportunities for teachers to listen to every student regularly helps them gauge where each student is in terms of mastering skills as well as how they are thinking about the problems. It is also important for teachers to focus on pertinent data that contributes to student growth, such as observing the variety of strategies employed by students in fluency development. Through effective questioning techniques, teachers can gauge students' understanding and subsequently guide them to achieve the mathematical objectives of the lesson (Roediger et al., 2011). For example, teachers can gauge things like whether students are leveraging their foundational facts and properties to figure out their remaining facts, using 5s and 2s to figure out 7s, or using the commutative property to make calculations easier for themselves. Without this crucial aspect of arithmetical reasoning by using principles such as commutativity, associativity and the addition/subtraction inverse principle, students will depend entirely on the facts that they have already learned, and will be unable to go beyond them independently (Dowker, 2009).

Building Fact Fluency helps teachers be thoughtful about how they are measuring growth and progress. Building Fact Fluency incorporates multiple opportunities for educators to assess this information in ways that honor students, inform teaching, and maintain focus on what really matters. Educators will notice that Building Fact Fluency does not include built-in summative assessments. Because the goal of Building Fact Fluency is getting students to know and be able to use their facts, the authentic measure of this ability is whether they readily know and use facts while solving real-world problems. When students are math fact fluent, they should engage in fluency actions, including: selecting appropriate strategies, solving problems in a reasonable amount of time, adapting strategies, applying strategies to new problem types, completing steps accurately, producing correct answers, and checking for reasonableness (Bay-Williams & SanGiovanni, 2021). Aligning to this goal of supporting authentic fact fluency, Zaner-Bloser Building Fact Fluency includes varied assessment opportunities such as observations, student interviews, student work, student journals, and student self-assessments. By providing multiple means of assessment, including student self-assessment, the program also helps to address historical disparities in data on fact fluency, which often exclude marginalized students, such as multilingual learners, from challenging grade-level mathematics (Huinker et al., 2020).

As a result of the consistent implementation of Building Fact Fluency, educators will be able to confidently tell that the instruction is working based on (1) the aforementioned formative assessment approaches, and (2) the summative measures that are already being implemented in the classroom.

Short-term Outcomes

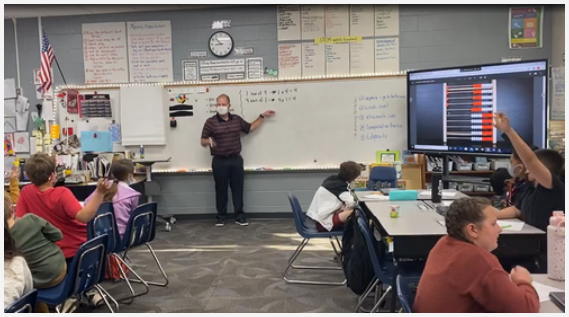
In the short-term, the outcomes of Building Fact Fluency are anticipated to be transformative for both teachers and students. By engaging with Zaner-Bloser Building Fact Fluency, students will demonstrate improved math fluency and more flexible application of strategic thinking, reduced math anxiety, and increased motivation to learn and build math skills. Students will approach new problems with multiple strategies as the program provides opportunities for students to exercise autonomy, relate to authentic contexts, and learn in a community in which they share with and learn from each other. As they engage in math discourse, they are building language and math understanding.

Teachers, through integrated professional learning opportunities, as well as implementation of Building Fact Fluency in the classroom, build their capacity in math instruction, successfully differentiating instruction for all learners and supporting student growth. Teachers will experience and exercise autonomy and flexibility so that the toolkit can be used in multiple grades as well as in a variety of settings.

Long-term Outcomes

Over time, Building Fact Fluency builds math equity by increasing access and removing barriers to engagement in rich math and building community, curiosity, and confidence in learning advanced math skills. Building Fact Fluency was designed as a resource that would disrupt the pattern of fact fluency as a gatekeeper, allowing all students to be taught math facts through engagement in rich math. By intentionally creating a program that reduces text load, provides images and videos to help develop language, and eliminates time pressure, the authors sought to create an inclusive environment where every student feels empowered to participate actively in their mathematical learning. Students develop relational thinking that applies to many aspects of their lives and can be built upon as they progress through higher level mathematics. Teachers and parents will notice that students become more creative and tenacious in solving problems and are more likely to feel comfortable engaging with the math community. As students become more adept and confident in problem-solving, they are not only acquiring mathematical skills but are also developing a strengthened mindset of math self-efficacy and ownership that is crucial for future success, not only in the math classroom, but across the learning spectrum.

Image 4. Example of hands-on, collaborative math instruction



- What do you notice? What do you wonder?
- How many bikes can he make?
- Estimate. What is your strategic choice of a number? What's the strategy behind your estimate?

How Does Zaner-Bloser Building Fact Fluency Help Students Learn?

Engaging Students in Learning

Why does engagement in math learning matter?

Active involvement in acquiring foundational math skills is paramount for nurturing positive attitudes, intrinsic motivation, and a deep understanding of mathematical concepts (Skinner et al., 2008). This engagement encompasses emotional investment, such as curiosity and enthusiasm, along with active participation, leading to enhanced retention and practical application of knowledge (Snyder et al., 2017). Engagement in learning involves students recognizing their capabilities, forming emotional connections with their coursework, and taking on proactive roles in their learning endeavors (Nayir, 2015; Moenikia & Zahed-Babelan, 2010). Research suggests that heightened engagement correlates with increased receptiveness to learning, indicating that students who are more engaged are more open to acquiring new knowledge and skills (Moenikia & Zahed-Babelan, 2010). Studies have found that children demonstrating higher levels of engagement tend to receive more support from their instructors; conversely, disengaged students may experience a reduction in support or an increase in control from their instructors over time (Skinner et al., 2008). Zaner-Bloser Building Fact Fluency not only equips educators with techniques to build math fluency but also offers a variety of tools aimed at boosting student engagement and enhancing learning outcomes.

How does Zaner-Bloser Building Fact Fluency engage students in learning?

Building Fact Fluency engages students by providing purposeful practice through meaningful real-world context, inquiry-based learning, and hands-on activities. The approach embeds opportunities for students to explore the conceptual understanding of basic facts and apply a range of strategies to rich problem-solving opportunities that pique their interest and invite flexible thinking and peer collaboration. The toolkit is designed to center meaningful and accessible contexts that leverage experiences that students can relate to and showcase the relevance of math beyond the classroom. Relevant and realistic contexts help students visualize the concepts and bring purpose to math learning. Additionally, Building Fact Fluency engages students through collaborative and hands-on learning activities including group discussions, collaborative tasks, and games where students work together on shared problems and access interactive, enjoyable avenues for practice.

DEEP DIVE

Building Fact Fluency incorporates meaningful, real-world contexts into math learning. The lessons are organized to introduce strategies designed to develop mathematical reasoning through a series of routines, activities, and Lesson Strings purposefully connected via real-world context. Practice should not be a 'meaningless drill' but should occur in a context of making sense of situations and number relationships (Clements et al., 2014). For example, the toolkit explores the "10 and Some More" strategy for addition/subtraction within contexts like a full box of 10 markers and some loose ones, a a whole pizza cut into ten slices with extras, and a dime and some pennies - all familiar to students. For multiplication/division, students think about groups of 4s with peaches in baskets, rows of coins, and basketballs on shelves.

Having contextually relevant activities enables students to experience authentic, realistic applications of the math reasoning they are learning, making learning interesting, engaging, and relevant. Within these meaningful contexts, students engage with vibrant images or videos and rich collaborative discussions. When students can relate to the learning environment, they feel a greater sense of belonging and increase engagement with the activities (Deci et al., 1999). Building Fact Fluency uses photographs, providing an additional visual cue for the authenticity of the problems. For example, Image Talks provide short routines based on photographs of everyday objects. The 3-Act Math Tasks are problem-based lessons broken into 3 parts based on intriguing visuals that foster engagement by sparking students' curiosity and inviting them into the mathematical conversation.

Building Fact Fluency also has several contextualized practice problems that provide collections of story problems directly related to the context. By design, these problems are specifically tied to the strategy students are learning, encouraging them to build connections and discuss their thinking. Students are actively engaged in making sense of story problems situated in the meaningful context by visualizing the scenario and reflecting on the actions and relationships. They are given an opportunity to practice representing their thinking in words, numbers, and pictures. Immersing students in these relevant contexts deepens their engagement and understanding.

> "What is important is that the contexts of tasks allow all students entry into the situation and allow all students to problematize the situations."
>
> -Hiebert et al. (1997), *Making Sense: Teaching and Learning Mathematics with Understanding*

Image 5. Example of contexts used for Addition/Subtraction

Strategy		Context	
Sums within 5	Crayons	Marbles	Toy cars
Plus and minus 0,1,2	Shells	Bears	Blocks
Combination for 10	Buttons	Markers	Peppers
10 and some more	Markers	Pizza	Coins
Doubles	Lemonade	Apples	Eggs
Near doubles	Apples	Eggs	Chopsticks
Pretend-10/ make-10	Markers	Pizza	Tennis balls

Building Fact Fluency uses **routines and activities** in Lesson Strings to provide many opportunities for engaging students in thoughtful reflection individually and with peers **through inquiry** designed to build their relational thinking of how the numbers are related. Through this guided inquiry, students connect classroom concepts to authentic real-world applications in a meaningful way. This process fosters higher-order thinking skills, allowing students to develop innovative strategies for constructing understanding, deriving meaning, and exercising creativity (Kuhlthau et al., 2015). Zaner-Bloser Building Fact Fluency provides students with regular opportunities to engage in dialogue and discussion to make thinking visible and meaningfully solve problems rather than memorize facts without understanding number relationships. Activities such as Image Talks encourage teachers to ask students what they notice and wonder, share those insights with each other, and then extend conversations by posing questions that facilitate thinking about the patterns and relationships reflected in the images. Similarly, Tool Talks invite students to discuss what they notice, wonder, and see happening in images that feature mathematical tools. This approach of immersing students in rich contexts while promoting inquiry and discourse deepens their engagement, relational understanding, and ability to innovate with numbers.

Building Fact Fluency also underscores the significance of cultivating engaging learning experiences for students by designing **hands-on, collaborative** math instruction. Hands-on experimental tasks in mathematics foster a positive attitude, encouraging active participation and independent comprehension. This transition moves students from being passive recipients to proactive learners (Eyrikh et al., 2019). In the design of Building Fact Fluency, meticulous attention was dedicated to crafting ample opportunities for hands-on learning to practice strategic thinking and meaningful application of strategies. Educational games play a crucial role in developing fact fluency by offering interactive and immersive experiences that reinforce mathematical concepts (Gray, 2013). Numerous games, such as Number Match for addition/subtraction, and Multiplication Leapfrog for multiplication/division fluency, are included in both core and optional routines. These routines offer a variety of board games and flexible applications for small and larger teams. Building Fact Fluency's toolkits are versatile and can be personalized to fit the needs of individual classrooms. Teachers can incorporate additional hands-on math tools within the activities.

Image 6. Sequence of factors and contexts

Sequence	Factor	Lesson String	Core Game
1	2s	Toy Bikes	Multiplication Cover Up with 2s
2	10s	Bowling	Choosing 5s or 10s
3	5s	Grapes	The Friendly Factor Game with 5s
4	Squares & Near Squares	Sushi	Building Squares*
5	2s	Honey Bears	Capturing Rectangles (with Side Lengths of 2)
6	4s	Peaches	The Double Double Game*
7	10s	Pencils	Collecting Quotients (Dividing by 10)
8	2s	Shoes	The Friendly Factor Game with 2s
9	3s	Tennis Balls	Multiplication Cover Up with 3s
10	5s	Crayons	Capturing Rectangles (with Side Lengths of 5)
11	6s	Eggs	Multiplication Cover Up with 6s
12	10s	Bracelets	The Friendly Factor Game with 10s
13	9s	Stickers	Multiplication Cover Up with 9s

Deepening Students' Math Fact Fluency

Why does deepening math fact fluency matter?

Mathematical fluency with basic facts lays the groundwork for later mathematical education. Proficiency in mathematics means that students grasp key mathematical concepts and effortlessly recall arithmetic facts (National Mathematics Advisory Panel [NMAP], 2008). Unfortunately, many students nationwide struggle with fundamental math skills (NAEP, 2022). While various factors contribute to math deficiencies, inadequate computational fluency is particularly influential (Kilpatrick et al., 2001; NMAP, 2008). Specifically, children lacking fluency in basic computations are at risk for math difficulties that persist into adulthood (Rivera & Bryant, 1992; Woodward, 2006). Moreover, students who can easily store and retrieve basic math facts are better positioned to develop problem-solving skills and understand abstract mathematical principles (Patton et al., 1997; Shapiro, 2010; Siegler & Shrager, 1984). Applying math facts to solve problems is a fundamental objective in math education (NMAP, 2008) and represents a distinct stage of understanding in cognitive learning models (Bloom et al., 1956). According to Bloom et al. (1956), application of a concept involves the "ability to apply the appropriate abstraction without having to be shown how to use it in that situation" (p. 120).

> "Psychologists have long known that people more easily learn a body of knowledge by focusing on its structure (i.e., underlying patterns and relationships) than by memorizing individual facts by rote. Furthermore, psychologists have long known that well-connected factual knowledge is easier to retain in memory and to transfer to other learning other new but related facts than are isolated facts."
>
> -- Baroody, A. J. (2006). *Why children have difficulties mastering the basic number combinations and how to help them.*

Deepening knowledge ensures the transfer of knowledge. "Deeper learning" can be described as the ability to apply knowledge to new situations or contexts, as well as to use that knowledge in entirely different application settings. This process often involves shared learning and interactions within a community, leading to expertise in a specific domain (National Research Council, 2012). Deeper learning yields transferable knowledge, including content knowledge and the understanding of how, why, and when to apply it to solve problems (National Research Council, 2012). An instructional approach that encourages students to delve into the conceptual understanding of fundamental facts, explore diverse methods to bolster their grasp of numerical concepts,

and subsequently participate in strategic practice to effortlessly recall information, fosters a robust and comprehensive learning foundation for students. (O'Connell et al, 2011) By integrating fact retrieval with conceptual understanding, barriers to math fluency are dismantled, opening avenues to enriched mathematical experiences. Building Fact Fluency offers strategies to foster deep conceptual understanding, thereby facilitating procedural fluency and fortifying foundational knowledge, ensuring seamless knowledge transfer.

How does Building Fact Fluency support students' deep understanding, transfer, and extension of math skills?

Building Fact Fluency is designed to support students' development of conceptual understanding, not just memorization, as a route to accurate, efficient, and flexible fact fluency. The instructional approach introduces students to strategies across meaningful contexts to build deeper understanding of relationships among number facts, facilitating procedural fluency as well as strengthening important foundational knowledge for more complex math learning. Strategy learning enables students to transfer their learning to new contexts and in a manner that is personalized, as they utilize strategies that work best for them. Lesson Strings include clusters of related tasks and activities that center strategy learning and flexible application to problem solving with many opportunities for students to discuss strategies, find patterns, and build connections across ideas. By incorporating interleaved and spaced practice methods that allow students to practice and revisit different strategies over time, the toolkit aligns with cognitive science principles, supporting long-term retention and mastery. Moreover, designed to disrupt and replace inequitable practices, the approach removes barriers to students' access to building deep mathematical understanding.

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Building Fact Fluency program prioritizes deep conceptual understanding and strategic thinking to ensure students not only grasp math facts but also apply them effectively. Research suggests that nurturing conceptual understanding early in elementary education fosters significant improvements in math proficiency. Tailored interventions, addressing both procedural and conceptual knowledge gaps, prove more effective in enhancing math skills (Burns et al., 2015). The Lesson Strings incorporated

in the program are designed to deepen students' knowledge of relationships between facts so that they can become not only accurate but can also be efficient and flexible.

Lesson Strings allow teachers to structure learning opportunities gradually, moving students' thinking along a continuum from counting strategies to additive thinking with the addition/subtraction toolkit and multiplicative reasoning with the multiplication/division toolkit. There are several core and optional routines, games, and activities included in the Lesson Strings that all promote and reinforce relational thinking, and strategy use. Building Fact Fluency employs **visualizations within familiar contexts** to foster deep conceptual understanding. Grounding mathematical concepts in familiar imagery enhances comprehension (Furner et al., 2005), facilitating students' grasp of these concepts more effectively. Within the program there are activities that deepen students' understanding by promoting visualization first with concrete visuals and contexts during 3-Act Math Tasks, Image Talks and Contextualized Practice Problems. Then, students apply strategic thinking and leverage the visualizations they built when the context is removed in Tool Talks and Number Talks. Students learn and practice addition/subtraction and multiplication/division fact strategies within accessible, visual, contextual themes. The familiarity of the visualizations in the context makes the learning "stick." Students build a repertoire of strategies to use as they work through the core and optional routines within the lesson strings. Having access to a range of strategies and having opportunities to reflect and make decisions about their strategy use builds math problem-solving skills they can transfer to new situations.

Image 7. Example of the lesson string in a context

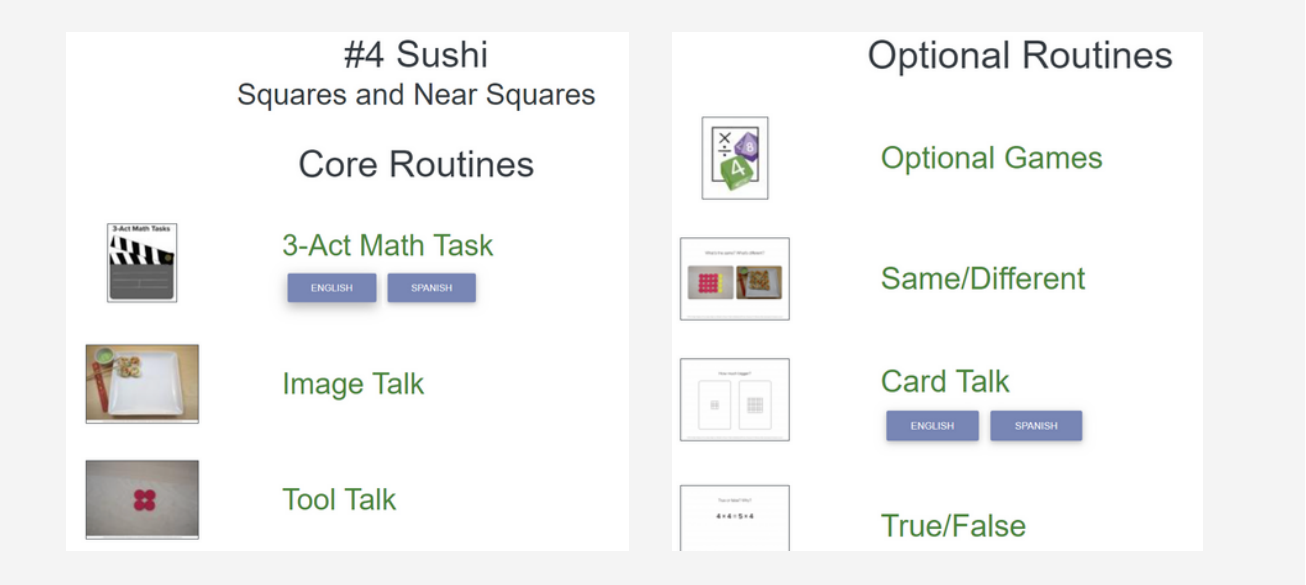
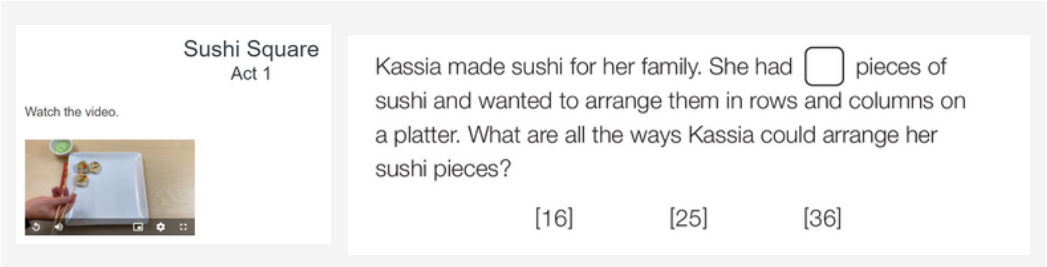


Image 8. Example of familiar contexts used within lesson strings

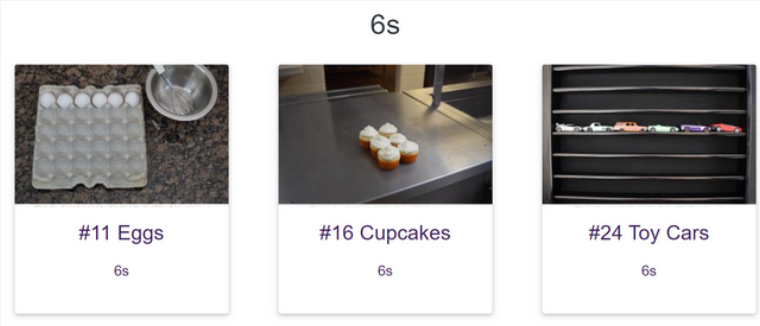


Building Fact Fluency incorporates **interleaved and spaced practice** methods to enhance learning and improve retention, transfer, and long-term mastery. The toolkit is structured to help teachers sequence and spiral learning contexts throughout the year, facilitating retrieval practice and reinforcing prior knowledge.

Strategically mixing different types of problems or previous concepts with newly introduced concepts is also known as interleaved practice. When concepts are interleaved, students engage in what's called "retrieval practice," which involves activating prior knowledge and practicing what they know which can significantly increase math learning and retention (Agarwal & Agostinelli, 2020; Rohrer et al., 2020). For example, in the Multiplication & Division Toolkit, students initially learn about groups of 4s using the "peaches" context. They then revisit this concept and discover how 4s are represented in arrays, using the "coins" context. Finally, they explore the three-dimensional representation of 4s using the "basketballs" context. Along with this interleaving of old and new skill practice, Building Fact Fluency also incorporates spaced practice by structuring activities to have deliberate intervals between repetitions of specific numerical skills.

In contrast to massed, or blocked practice, where concepts are introduced and practiced in a single condensed setting, spaced practice embeds intentional intervals between the practice of particular concepts, providing repeated practice across longer time frames.Spaced practice has been shown to promote mastery learning and long-term retention to a much greater extent than massed practice (Brown et al., 2014).

Image 9. Example of spaced practice of factor 6



In the Building Fact Fluency toolkit, instead of grouping each strategy or factor for a specific length of time, they are dispersed throughout the toolkits. For instance, in the Multiplication & Division Toolkit, the number 6 is introduced in the eleventh, sixteenth, and twenty-fourth context. Furthermore, Building Fact Fluency incorporates varied practice methods that encourage students to utilize various means and representations. These activities encompass practicing with or without contexts, tools, mental math, and written repetitions, engaging in discussions, playing games, solving various problem types, and working with different numbers. This mixed practice fosters productive struggle, aiding in the creation of more efficient neural pathways in the brain (Sriram, 2020). Ultimately, by employing interleaving and spaced repetition techniques, Building Fact Fluency ensures students develop a deep and lasting understanding of math concepts.

Building Fact Fluency is **designed to remove barriers** to building deep mathematical understanding. The approach prioritizes conceptual understanding and relational thinking over rote, decontextualized memorization, ensuring that all students can engage with rich mathematical instruction, regardless of their initial mastery of basic facts. Procedural fluency, which encompasses learning basic number combinations and formulas, starts with a strong foundation of deep conceptual understanding. This approach is crucial not only for fostering positive mathematical identities and agency but also as a prerequisite. Rushing children to memorize facts and formulas without sufficient understanding deprives them of opportunities to gain confidence in their mathematical abilities. Procedural fluency develops gradually over the years when elementary school mathematics instruction prioritizes understanding and sense-making of concepts (Huinker et al., 2020). The toolkit of activities is purposefully designed to **enhance accessibility for a broad spectrum of learners.** In accommodating diverse learners, a multimodal approach proves advantageous in facilitating learning (Furner et al., 2005). Research suggests that multimedia learning environments with carefully designed multimodal interactions can promote children's conceptual learning by providing a richer perceptual experience (Paek, 2012). Building Fact Fluency is multimodal, providing diverse practice applications, and utilizing vibrant and authentic visuals to build math language. Additionally, contexts and situations are chosen to resonate with students' experiences, enhancing their ability to relate to the material. Furthermore, time pressures, such as timed drills and tests, are eliminated to create a more conducive learning environment. Building Fact Fluency provides a holistic structure that aids students in cultivating deep comprehension while also enabling the application and expansion of mathematical skills, thereby establishing a strong basis for mathematical proficiency.

Image 10. Examples of contexts used to teach strategies

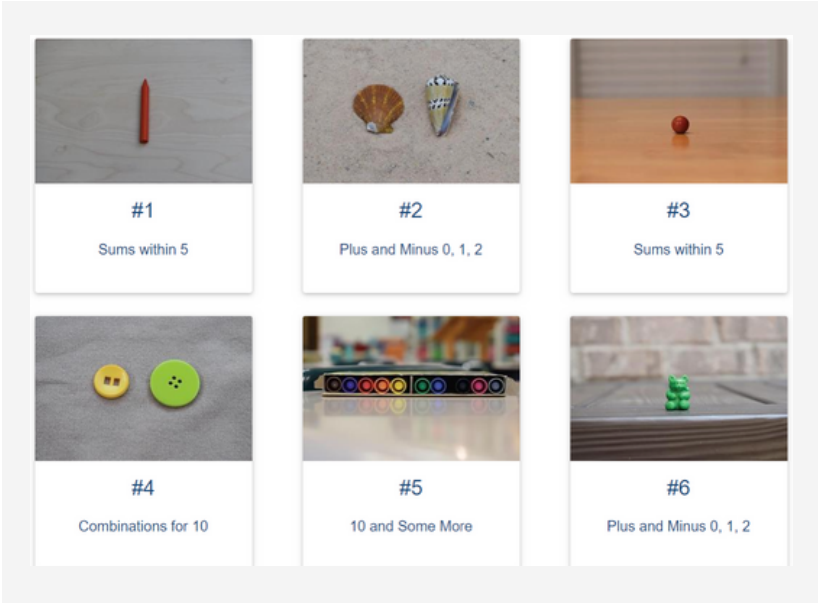
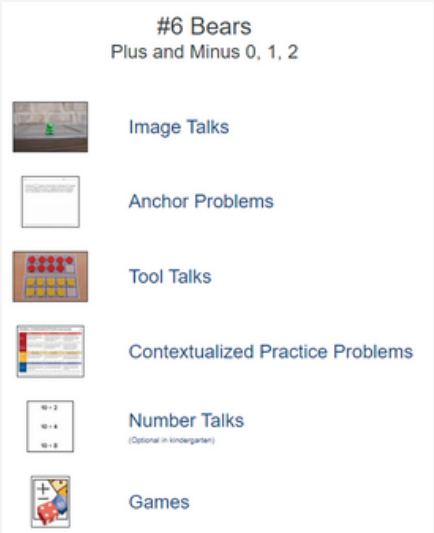


Image 11. Examples of diverse practice application



Strengthening Students' Ownership of Math Learning

Why does a student's ownership of math matter?

A student's sense of ownership over mathematics encompasses not only their personal beliefs about their abilities as math learners but also how they perceive others' perceptions of their mathematical prowess and their understanding of the nature of mathematical abilities (Solomon, 2009). This ownership is pivotal in shaping the comprehensive set of skills, attitudes, and relationships required for students to excel in their mathematical learning journey (Aguirre et al., 2013). It involves the cultivation of dispositions and deeply held beliefs that empower students to effectively engage with and apply mathematics in various contexts throughout their lives (Aguirre et al., 2013). Math anxiety is a prevalent issue among adults, and its interaction with variables such as self-efficacy and motivation can either exacerbate or alleviate this anxiety (Luttenberger et al., 2018). Particularly during the formative years when students are developing their identities and taking ownership of their mathematical learning, negative perceptions of math, perpetuation

> "How mathematical proficiency is defined and communicated to students has a powerful impact on their mathematics identities and their exercise of various forms of agency."
>
> -Aguirre, J et al. (2003). *The impact of identity in K-8 mathematics learning and teaching: rethinking equity-based practices.*

of myths, and strained relationships with math can significantly influence their academic and professional trajectories (Allen & Scnhell, 2016).

One research-based concept utilized in mathematics education to foster math ownership is the notion of "mindset" (Dweck, 2006). Individuals with a "growth mindset" perceive intelligence, abilities, and talents as malleable attributes that can be developed through persistent effort, whereas those with a "fixed mindset" view these qualities as inherent and unchangeable (Dweck, 2006). Dweck's research underscores the advantages of embracing a growth mindset across various domains, including math education. Students with a growth mindset typically outperform those with a fixed mindset, as they believe in the potential for improvement and growth (Blackwell, Trzesniewski, & Dweck, 2007; Claro, Paunesku, & Dweck, 2016). Teachers' instructional decisions play a crucial role in shaping each student's sense of ownership over mathematics, from the learning environments they cultivate to their daily teaching strategies.

Teachers possess the capacity to shape students' ownership of math and challenge prevalent societal myths about mathematics (Aguirre et al., 2013). Educators play a vital role in nurturing a growth mindset among students by guiding their language and actions to focus on the learning process rather than innate abilities, thus creating conditions conducive to the development of a growth mindset in children (Haimovitz & Dweck, 2017). Research indicates that teachers who promote a growth mindset prioritize understanding over correctness and encourage students to articulate their problem-solving approaches (Sun, 2015). With Zaner-Bloser Building Fact Fluency, educators can empower students with a flexible "toolbox" of strategies, enhancing self-efficacy and fostering a culture of inclusivity and confidence in mathematical discourse.

How does Building Fact Fluency strengthen math ownership?

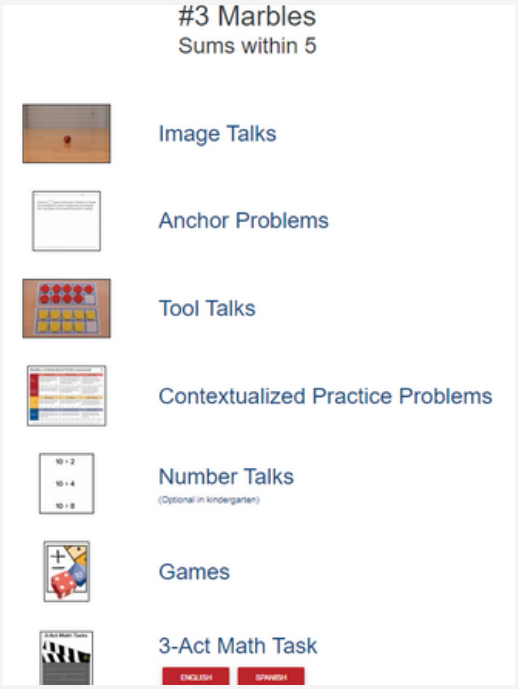
Zaner-Bloser Building Fact Fluency is designed to reinforce a broader view of math proficiency for educators and students by valuing the many ways students can arrive at answers to problems and offering multiple pathways and entry points to math proficiency. Lesson Strings include routines and activities that invite students to explore different strategies that are purposefully connected through context. Students are provided multiple strategies and are empowered to select strategies, which fosters their autonomy in math learning. Additionally, the approach intentionally centers discussion, dialogue, inquiry, and authentic contexts which build students' sense of belonging. Moreover, formative and diagnostic assessments are designed to encourage educators to sit

beside students and gather insights into student thinking at the moment rather than focus only on grading for accuracy. As valued, active participants in their learning, students gain ownership over their math learning, increasing their positive and growth-oriented view of themselves as math learners.

DEEP DIVE

In fostering a broad view of math proficiency, students are encouraged to explore multiple pathways to arrive at solutions. Research suggests that students who learn to employ various problem-solving strategies demonstrate enhanced ease and flexibility in tackling mathematical challenges (Woodward et al., 2018). Studies further support the notion that individuals adept at utilizing multiple strategies tend to achieve greater success in problem-solving endeavors (Dowker 1992; Siegler 2003). Regular exposure to diverse problem-solving scenarios cultivates a repertoire of strategies among students, enabling them to approach problems with increased efficiency and adaptability (Star and Rittle-Johnson, 2008). This multifaceted skill set empowers students to select the most appropriate strategies for each problem, ultimately facilitating smoother problem-solving experiences (Siegler, 2003). Zaner-Bloser Building Fact Fluency incorporates Lesson Strings, a series of interconnected activities, that foster visual and contextual understanding and empower students to cultivate diverse problem-solving strategies. By embracing students' varied approaches, the toolkit promotes a culture of inclusivity and flexibility in mathematical learning, enriching the educational experience for all.

Image 12. Example of diverse problem-solving scenarios within lesson strings

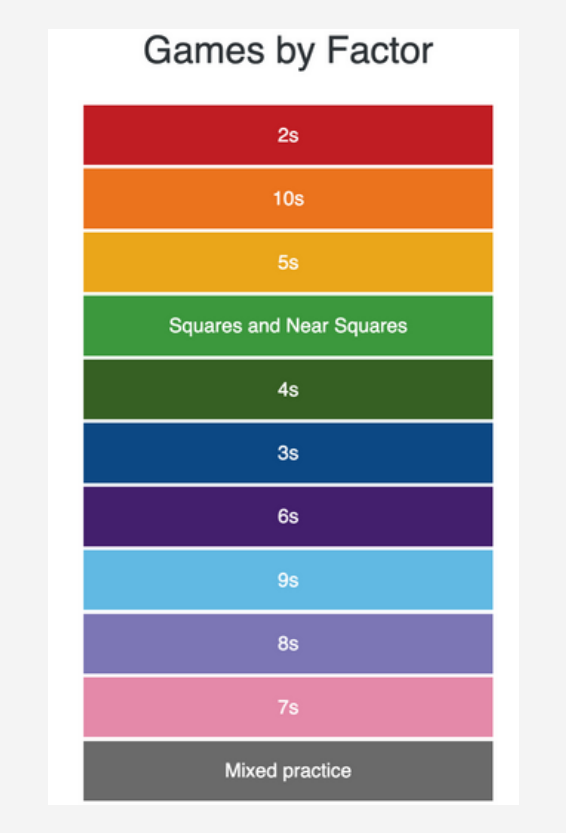


Zaner-Bloser Building Fact Fluency places a **strong emphasis on student autonomy and belonging.** Student autonomy encompasses situations where students are empowered to make decisions regarding their learning journey. Encouraging student autonomy in mathematics classrooms has been shown to enhance metacognitive skills and elevate academic performance (Kramarski & Michalsky, 2010). Research suggests that when

students can choose learning experiences tailored to their interests and needs, it can enhance engagement and enjoyment of the learning process. For instance, student autonomy within learning environments bolsters intrinsic motivation and academic achievement (Deci et al., 1999). Lesson Strings play a key role in fostering student autonomy and a sense of belonging. By offering routines and activities that empower students to select strategies tailored to their preferences and strengths, the toolkit cultivates a mindset of independence and self-reliance. Through this process, students learn to think flexibly and discern which strategies are most effective for them, thereby fostering a sense of ownership over their learning journey and contributing to a supportive classroom environment where every student feels valued and empowered.

Games are strategy-based and allow students opportunities to practice individual and mixed or varied facts. The games reinforce strategic thinking and allow for problem-solving practice and application opportunities. Students gain autonomy in their strategy selection and use it as they strengthen their strategic thinking. Math dialogue and discussions are another key strategy adopted to enhance student autonomy. Collaborative problem-solving activities play a crucial role in shaping individuals' understanding of mathematical concepts and building their identities within a learning community (Zack and Graves, 2001). Core routines center math discussion and dialogue, valuing student input and different paths to problem-solving. Research by Zack and Graves (2001) also suggests that the use of dialogue redefines class participation. Discussions and dialogue that are facilitated through the activities empower students to feel more comfortable with owning math conversation and sharing their strategies, thoughts, and ideas with each other.

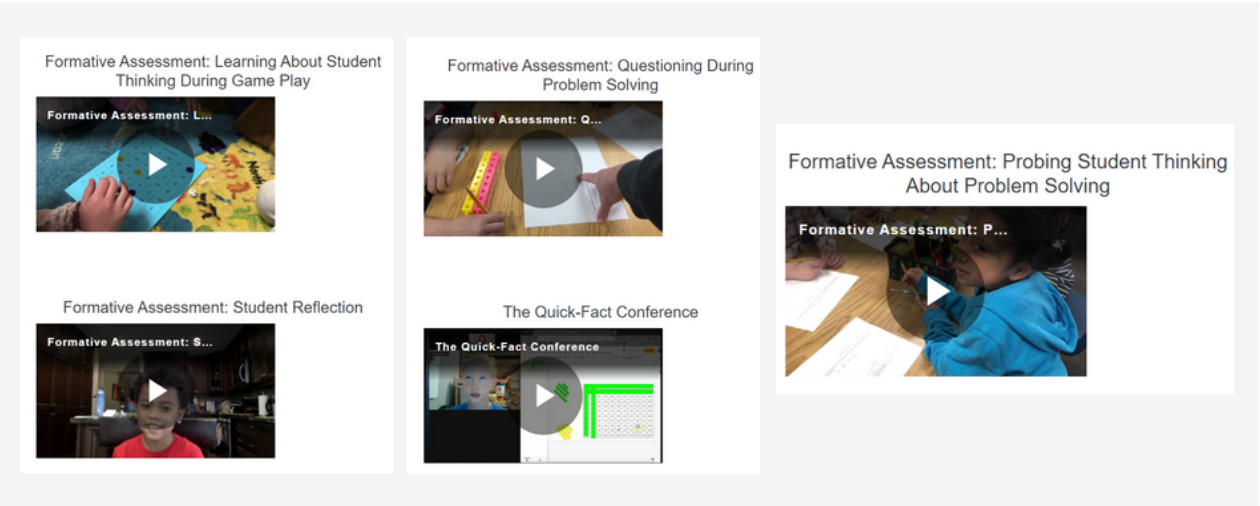
Image 13. Example of list of games by factors



Zaner-Bloser Building Fact Fluency utilizes Dynamic Assessment, which ensures learners are actively involved in their learning process. The data acquired through the dynamic assessment process offer teachers an extensive evaluation of their students' mathematical understandings and thinking which enables teachers to tailor their instruction to meet

the specific mathematical learning needs of their students (Allsopp et al., 2008). Throughout the toolkit, educators are encouraged to employ formative and diagnostic assessments that provide opportunities for students to discuss relational properties, listen to student thinking in real time, and use probes to guide students' thinking to the next level. Teachers who establish a growth mindset classroom focus on teaching for understanding, providing feedback that enhances understanding, offering opportunities for students to revise their work and demonstrate their growing understanding, and emphasizing the importance of effort and struggle in the learning process (Hooper et al., 2016). Active involvement in learning mathematics, coupled with supportive relationships and engaging experiences in the classroom, can lead to better performance and the development of a strong identity as a mathematics learner (Andreson, 2007).

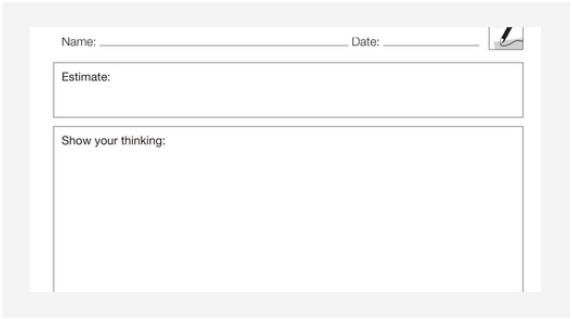
Image 14. Examples of different formative assessments teachers can utilize



Self-assessment is interwoven throughout Zaner-Bloser Building Fact Fluency. Students exercise autonomy by evaluating how their fluency is progressing and communicating their areas of strength and areas needing improvement. This hands students the reins of their learning process and aids in gradually mastering it (Gholami, 2016). Engaging in self-assessment and self-evaluation not only helps students evaluate their own progress but also fosters the development of communication skills and increased mathematics vocabulary (Stallings & Tascione, 1996). Within the toolkit, various mechanisms facilitate student reflection. There are student reflection questions after problem-based activities, allowing students to reflect and assess their own learning. This reflection and feedback mechanism enables students to contemplate their learning journey and identify areas for growth and improvement. Thought-provoking questions stimulate students' inquiries and connections to key concepts and examples. Educators can access a comprehensive list of reflection prompts as well as reflection questions seamlessly integrated within the 3-Act Math Task activities. Additionally, sample journal prompts and reflection sheets are provided to assist students in engaging

with self-assessment and reflection exercises effectively. This approach encourages students to become proactive participants in their learning journey, fostering a sense of ownership over their mathematical education. This approach also helps shape students' self perceptions as math learners, characterized by positivity and a growth-oriented mindset.

Image 15. Example of Self Reflection prompts for students



Conclusion

Building Fact Fluency energizes math education and increases students' procedural fluency by prioritizing the flexible use of math facts in authentic contexts, which is essential for math development. Rooted in research, this program meticulously progresses from concrete to abstract learning, engaging students with real-world activities. It employs evidence-based techniques like interleaved practice to enhance long-term retention, aligning with cognitive development for mastery.

Moreover, Building Fact Fluency fosters student autonomy through structured and predictable learning routines. Its assessment tools enable tailored instruction and progress tracking. In the short term, it boosts reasoning skills and confidence, while in the long term, it aims to cultivate enduring changes in students' math attitudes and approaches, important for lifelong mathematical success. Ultimately, it aims to deepen students' understanding of basic arithmetic operations by providing rich learning experiences that prioritize strategic thinking and flexibility. These experiences empower students to tackle increasingly complex mathematical challenges with curiosity and competence, paving the way for lifelong mathematical success in the 21st-century educational landscape.

Ready to improve your students' fact fluency and strategic thinking?

Use these questions to reflect on your current practice, considering the research featured in this paper.

- Are your students engaged in interactive and motivating activities that build fact fluency?
- Do your lessons provide an efficient way to help students access grade-level math?
- How do you monitor student progress in building fact fluency when they are learning operations (addition, subtraction, multiplication and division)?
- Are your students demonstrating flexibility in applying their math facts?
- Are your students provided with clear feedback and support to build fact fluency?
- Do your students have opportunities to practice their math skills across various contexts?

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