Building Fact Fluency

A TOOLKIT FOR MULTIPLICATION & DIVISION

By Graham Fletcher
& Tracy Johnston Zager

CORRELATION TO

Common Core State Standards for Mathematics

CONTENTS

Stenhouse PUBLISHERS

WWW.STENHOUSE.COM
800.988.9812

Common Core State Standards for Mathematics

– STANDARDS FOR MATHEMATICAL PRACTICE, APPLICABLE FOR ALL GRADES –

CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.	Embedded throughout *Building Fact Fluency,* especially in these routines: · Contextualized Practice Problems · 3-Act Math Tasks · Open Middle Problems · Journaling and Reflection
CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.	Embedded throughout *Building Fact Fluency,* especially in these routines: · True/False · Card Talks · Open Middle Problems · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · 3-Act Math Tasks · Games · Same/Different · Journaling and Reflection
CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.	Embedded throughout *Building Fact Fluency,* especially in problem-based lesson discussions, student representations, and the following routines: · True/False · Card Talks · Contextualized Practice Problems · 3-Act Math Tasks · Open Middle Problems · Same/Different · Image Talks · Tool Talks · Number Talks
CCSS.MATH.PRACTICE.MP4 Model with mathematics.	Embedded throughout *Building Fact Fluency,* especially in these problem-based routines: · 3-Act Math Tasks · Contextualized Practice Problems
CCSS.MATH.PRACTICE.MP5 Use appropriate tools strategically.	Embedded throughout *Building Fact Fluency,* especially in these routines: · Tool Talks · Card Talks · Games · Contextualized Practice Problems

CCSS.MATH.PRACTICE.MP6 Attend to precision.	Embedded throughout *Building Fact Fluency,* especially in these routines: · True/False · Card Talks · Same/Different · Open Middle Problems · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · 3-Act Math Tasks · Games · Journaling and Reflection
CCSS.MATH.PRACTICE.MP7 Look for and make use of structure.	Embedded throughout *Building Fact Fluency,* especially in these routines: · Card Talks · True/False · Same/Different · Image Talks · Tool Talks · Number Talks · Games · Journaling and Reflection
CCSS.MATH.PRACTICE.MP8 Look for and express regularity in repeated reasoning.	Embedded throughout *Building Fact Fluency,* especially in these routines: · Image Talks · Tool Talks · Number Talks · Open Middle Problems · Games · Open Middle Problems · True/False · Same/Different · Card Talks · Journaling and Reflection

Common Core State Standards for Mathematics

– GRADE 2 –

Work with equal groups of objects to gain foundations for multiplication.	
CCSS.MATH.CONTENT.2.OA.C.3 Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	The *Building Fact Fluency* toolkit extends the additive thinking that students have developed in K–2 classrooms. Students work with 2s extensively in the Toy Bikes, Honey Bears, and Shoes Lesson Strings, and extend patterns with even numbers through games such as the Double Double Game.
CCSS.MATH.CONTENT.2.OA.C.4 Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	The structure of rectangular arrays is a powerful model that is used throughout the toolkit. Many of the Image Talks and Tool Talks incorporate arrays as a way to model multiplication. Half of the Card Talks are specifically designed to explore the structure of arrays.

Common Core State Standards for Mathematics

– GRADE 3 –

Represent and solve problems involving multiplication and division.	
CCSS.MATH.CONTENT.3.OA.A.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*	The entire *Building Fact Fluency* toolkit aligns to this standard, especially the Image Talks, 3-Act Math Tasks, and Contextualized Practice Problems, in which students make sense of and talk about multiplication in everyday contexts. The Card Talks and Tool Talks support the use of mathematical tools and models to interpret and represent multiplication.
CCSS.MATH.CONTENT.3.OA.A.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*	Multiplication and division are taught in an intertwined manner in *Building Fact Fluency*, so students interpret numbers in both partitive and quotative division situations throughout the Lesson Strings and the toolkit. The 3-Act Math Tasks and Contextualized Practice Problems give students opportunities to reason about division in everyday contexts.
CCSS.MATH.CONTENT.3.OA.A.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]	The Contextualized Practice Problems provide regular opportunities for students to use multiplication and division in word problems. In 3-Act Math Tasks, students both pose and solve the word problems. The contexts incorporate grouping (e.g., Grapes, Apple Slices, Lemons), arrays (e.g., Sushi, Stickers, Coin Collection), and measurement quantities (e.g., Honey Bears, Origami, Bracelets).
CCSS.MATH.CONTENT.3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*	*Building Fact Fluency* was designed to emphasize the relationship between the operations of multiplication and division. Throughout the problems in the toolkit, the unknown number varies, so students have ample practice determining the number of groups, or the number of items in each group, or the number of items. The games provide meaningful practice for students to determine the unknown number in a range of different equations.

Understand properties of multiplication and the relationship between multiplication and division.	
CCSS.MATH.CONTENT.3.OA.B.5 Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*	Every single task in *Building Fact Fluency* aligns to this standard. For example, the series of problems in each Image, Tool, and Number Talk is sequenced to encourage students to notice and apply the properties (e.g., a picture of 5 boxes, each with 10 pencils, is followed by a picture of 10 boxes, each with 5 pencils). The True/False, Same/Different, and Card Talks provide opportunities to make the properties explicit and generalize from specific problems to larger truths about the operations. Contexts were carefully chosen to highlight all the properties of multiplication.
CCSS.MATH.CONTENT.3.OA.B.6 Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8*.	*Building Fact Fluency* was designed to emphasize the relationship between the operations of multiplication and division. Throughout the problems in the toolkit, the unknown number varies, so students have ample practice determining the number of groups, or the number of items in each group, or the number of items. Students are encouraged to write a variety of equations to represent these situations (e.g., they can write a division problem as either a division equation or a multiplication equation with a missing factor) and to use the properties of the operations to solve them.
Multiply and divide within 100.	
CCSS.MATH.CONTENT.3.OA.C.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	Every single task in *Building Fact Fluency* aligns to this standard.
Solve problems involving the four operations, and identify and explain patterns in arithmetic.	
CCSS.MATH.CONTENT.3.OA.D.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends*.	There are many opportunities for students to generate and notice patterns in multiples and factors in *Building Fact Fluency*. For example, teachers can ask students what they notice about the game boards for games that involve 2s, 4s, and 8s; 5s and 10s; or 3s, 6s, and 9s. Each Image, Tool, and Number Talk is sequenced to reveal patterns. The Open Middle Problems often invite students to notice and leverage patterns.

Use place value understanding and properties of operations to perform multi-digit arithmetic.	
CCSS.MATH.CONTENT.3.NBT.A.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.	The Contextualized Practice Problems always include a multidigit option, and problems often involve multiples of 10. The Range Game with 10s provides explicit practice with this standard.
Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	
CCSS.MATH.CONTENT.3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.	Several of the Lesson Strings, such as Stickers and Origami, develop foundational concepts about area measurement.
CCSS.MATH.CONTENT.3.MD.C.5.A A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.	The area work in *Building Fact Fluency* is primarily designed to build understanding of multiplication rather than measurement, but the Stickers and Origami contexts both develop the idea of unit squares that have one square unit of area. In addition, several Tool Talks involve covering graph paper with unit squares.
CCSS.MATH.CONTENT.3.MD.C.5.B A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.	Covering a surface by tiling with unit squares without gaps or overlaps is emphasized in the Stickers Lesson String, including the 3-Act Math Task, and also reappears in several Tool Talks.
CCSS.MATH.CONTENT.3.MD.C.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	The Tool Talks featuring wooden cubes on graph paper are well suited to this standard.
CCSS.MATH.CONTENT.3.MD.C.7 Relate area to the operations of multiplication and addition.	Area is an outgrowth of earlier array work in *Building Fact Fluency*. Students will have opportunities to build conceptual understanding of area through multiplication.
CCSS.MATH.CONTENT.3.MD.C.7.A Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.	The foundations of this standard are found in several Lesson Strings, and the relationship between the product of the side lengths and the area of a rectangle is most explicit in Origami.

CCSS.MATH.CONTENT.3.MD.C.7.B Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	The everyday contexts in *Building Fact Fluency* invite students to build their understanding of area, side lengths, and multiplication from the outset.
CCSS.MATH.CONTENT.3.MD.C.7.C Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.	Multiple array contexts were designed to invite this type of thinking: Eggs, Paints, Coin Collection, Emojis, Balloons, Sushi, Golf Balls, and multiple Tool Talks. Students can extend this deep understanding of arrays and the distributive property to area in both Stickers and Origami. In addition, the multi-digit Contextualized Practice Problems invite use of area models.
CCSS.MATH.CONTENT.3.MD.C.7.D Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	Multiple array contexts invite decomposing rectangles of everyday objects into smaller arrays to solve. This array work lays the foundation of this standard, and the resulting conceptual understanding can be extended to area.

Common Core State Standards for Mathematics

– GRADE 4 –

Use the four operations with whole numbers to solve problems.	
CCSS.MATH.CONTENT.4.OA.A.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	Multiple Lesson Strings were designed to invite multiplicative comparison, particularly Honey Bears and Stacking Blocks. Comparisons are also interwoven throughout *Building Fact Fluency* in the Contextualized Practice Problems.
CCSS.MATH.CONTENT.4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]	Students have multiple opportunities to solve, represent, and discuss word problems involving multiplicative comparison. Honey Bears and Stacking Blocks focus on multiplicative comparison of liquid ounces and tower height, respectively, but comparison problems are peppered throughout the Contextualized Practice Problems.
CCSS.MATH.CONTENT.4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Multistep word problems are beyond the scope of *Building Fact Fluency* but interpreting leftovers and remainders in and out of context is included. Contextualized Practice Problems sometimes involve remainders, especially in Set B, and the game "Capturing Leftovers" explicitly focuses on them. 3-Act Math Tasks include estimation as part of the problem-solving process, with students invited to revisit their estimates after solving to assess reasonableness of their answers and effectiveness of their estimation techniques.
Gain familiarity with factors and multiples.	
CCSS.MATH.CONTENT.4.OA.B.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	Many *Building Fact Fluency* games invite students to notice patterns in factors and multiples, and to look for relationships among them. For example, the game Triple Play involves multiples of 3s, 6s, and 9s. The Double Double Double Game will help students notice relationships among the multiples of 2, 4, and 8. The game Choosing 5s or 10s builds the relationship between these two important factors. Work with the Array Cards is particularly helpful for building understanding of prime and composite numbers.

Generate and analyze patterns.	
CCSS.MATH.CONTENT.4.OA.C.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way*.	While students are not asked to generate patterns in *Building Fact Fluency*, the Image Talks, Tool Talks, and Number Talks are all composed of series of problems that relate in some way. As teachers record solutions to the string of problems, students will notice, question, and extend the patterns that emerge.
Use place value understanding and properties of operations to perform multi-digit arithmetic.	
CCSS.MATH.CONTENT.4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	The focus of *Building Fact Fluency* is single-digit multiplication and the related division facts, but there are multidigit problems (generally one-digit by two-digit but a few bigger numbers) in the Contextualized Practice Problems and 3-Act Math Tasks. Property-based explanations using equations, arrays, and area models are foundational to the toolkit, however, and frequent work with these representations should support students' conceptual understanding of multi-digit arithmetic in core curriculum.
CCSS.MATH.CONTENT.4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	The numbers in *Building Fact Fluency* are smaller than four-digit divided by one-digit, but there are many opportunities for students to develop division strategies based on place value, the properties, and the relationship between multiplication and division. Students are frequently expected to represent and explain their work with equations, arrays, and area models. Remainders appear in Contextualized Practice Problems and the game Capturing Leftovers.

Common Core State Standards for Mathematics

– GRADE 5 –

Write and interpret numerical expressions.	
CCSS.MATH.CONTENT.5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Because of the emphasis on properties, parentheses are used frequently in *Building Fact Fluency*. Both the True/False and the Open Middle Problem routines invite students to use and understand these symbols. Teachers can also model them while annotating Image, Tool, Number, and Card Talks.
CCSS.MATH.CONTENT.5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*	The True/False routine explicitly encourages students to interpret numerical expressions and reason about them without computing. For example, considering whether (10 × 11) ÷ 2 = 5 × 11 or 6 × 15 = 5 × 16 are true or false without evaluating will encourage algebraic reasoning and more general thinking about the operations.
Analyze patterns and relationships.	
CCSS.MATH.CONTENT.5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*	Many of the Open Middle problems ask students to analyze the patterns and relationships through the use of an input/output table.
Geometric measurement: understand concepts of volume.	
CCSS.MATH.CONTENT.5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.	Volume is not explicitly addressed in *Building Fact Fluency*, but many foundational concepts of volume are introduced through contexts that invite filling, stacking, or packing a rectangular prism, such as Stacking Blocks, Crayons, Pencils, and Basketballs, and in the Tool Talks involving rectangular prisms of Snap Cubes.

CCSS.MATH.CONTENT.5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.	Volume is not explicitly addressed in *Building Fact Fluency*, but many foundational concepts of volume are introduced through contexts that invite filling, stacking, or packing a rectangular prism, such as Stacking Blocks, Crayons, Pencils, and Basketballs, and in the Tool Talks involving rectangular prisms of Snap Cubes. In these contexts, students will naturally talk about *How many are in one slice (or layer)? How many slices (or layers) are there?*
CCSS.MATH.CONTENT.5.MD.C.5.A Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.	Volume is not explicitly addressed in *Building Fact Fluency*, but many foundational concepts of volume are introduced through contexts that invite filling, stacking, or packing a rectangular prism, such as Stacking Blocks, Crayons, Pencils, and Basketballs, and in the Tool Talks involving rectangular prisms of Snap Cubes. These tasks are explicitly designed to encourage conversation about the associative property of multiplication. Students will discuss how the quantity of blocks (or cubes, packs of crayons, etc.) is the same regardless of which base they multiply first, or which instance of multiplication they compute first.

WWW.STENHOUSE.COM
800.988.9812